

Unit 7 Obj. 7

We're reaching the end of the unit. Today's lesson will add just a few more concepts to what you've already learned. One of the more common questions at this point is "when do I need to complete the square?" The answer is this:

If you can tell what the vertex (parabola) or center (circle) is, then you don't need to complete the square. It's done. If you can't, then you need to complete the square.

For examples 1-3:

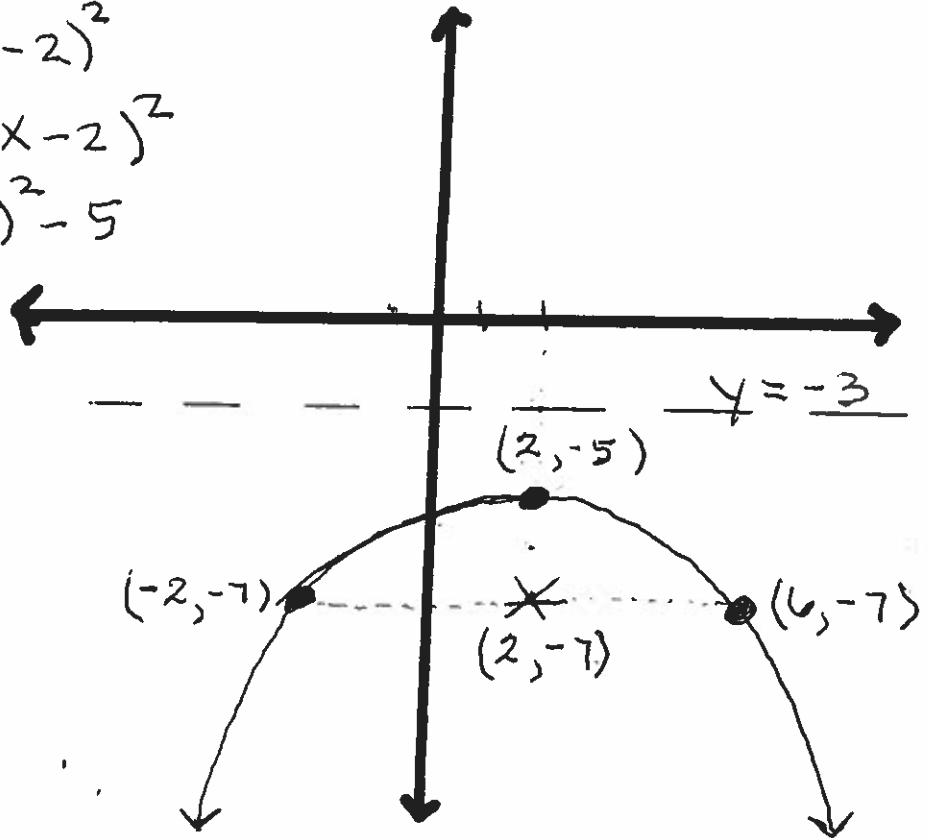
Sketch the parabola, identify the vertex, focus, directrix, focal chord endpoints, the axis of symmetry and the number of "x" and "y" intercepts.

Example #1: $8(y + 5) + (x - 2)^2 = 0$

$$8(y + 5) = -(x - 2)^2$$

$$y + 5 = -\frac{1}{8}(x - 2)^2$$

$$y = -\frac{1}{8}(x - 2)^2 - 5$$



vertex: $(2, -5)$

$$c = \frac{1}{4a} \quad a = -\frac{1}{8}$$

$$c = \frac{1}{4(-\frac{1}{8})}$$

$$c = \frac{1}{-\frac{4}{8}} = \frac{1}{-\frac{1}{2}} = -2$$

$c = -2$

$2c = 4$

Focus = $(2, -7)$ Dir = $y = -3$

$(-2, -7)$ $(6, -7)$ Focal Chord Endpoints

x intercepts = 0 y intercepts = 1

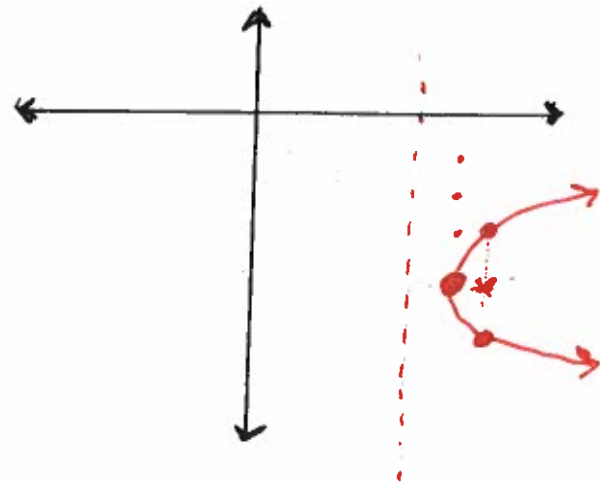
Example #2: $x = 2y^2 + 16y + 99$



$$\begin{aligned}x - 99 &= 2y^2 + 16y \\x - 99 &= 2(y^2 + 8y + \frac{16}{2}) \\&\quad + 32 \qquad \qquad \downarrow \\&\qquad \qquad \qquad (4)^2 \nearrow\end{aligned}$$

$$\begin{aligned}x - 67 &= 2(y + 4)^2 \\&\quad + 67 \qquad \qquad \qquad + 67\end{aligned}$$

$$\boxed{x = 2(y + 4)^2 + 67}$$



Vertex: $(67, -4)$

Focus: $(67\frac{1}{8}, -4)$

$$c = \frac{1}{4a} = \frac{1}{4(2)} = \frac{1}{8}$$

Directrix: $x = 66\frac{7}{8}$

$$\boxed{c = \frac{1}{8}}$$

Focal chord Endpoints:

$$2c = \frac{2}{8} = \frac{1}{4}$$

$(67\frac{1}{8}, -4\frac{1}{4})$ $(67\frac{1}{8}, -3\frac{3}{4})$

$$\boxed{2c = \frac{1}{4}}$$

x-intercepts 1

y-intercepts 0

Example #3: $y + 40 = -x^2 + 24x$ ↻

$$y + 40 = -1 \left(x^2 - 24x + \frac{144}{(-12)^2} \right) - 144$$

$$y - 104 = -1(x - 12)^2$$

$$y = -1(x - 12)^2 + 104$$

Vertex: $(12, 104)$

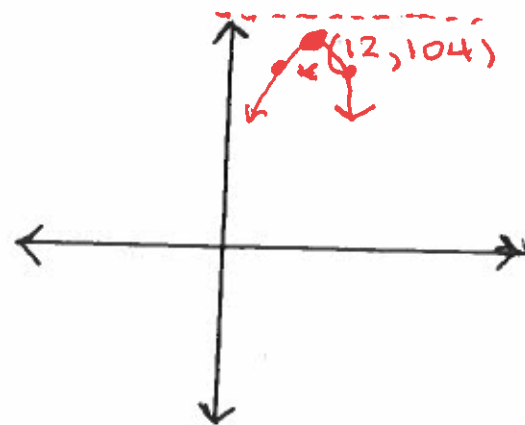
$$c = \frac{1}{4a}$$

$$c = \frac{1}{4(-1)} = -\frac{1}{4} = -\frac{1}{4}$$

$$c = -\frac{1}{4}$$

$$2c = 2 \cdot -\frac{1}{4} = -\frac{2}{4} = -\frac{1}{2}$$

$$2c = \frac{1}{2}$$



Focus: $(12, 103\frac{3}{4})$

Directrix: $y = 104\frac{1}{4}$

Focal Chord Endpoints:

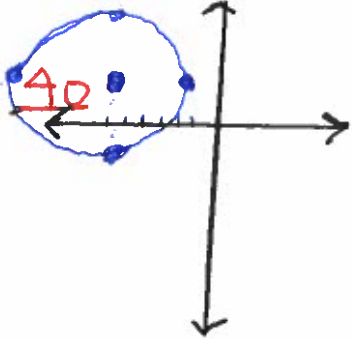
$$(11\frac{1}{2}, 103\frac{3}{4}) (12\frac{1}{2}, 103\frac{3}{4})$$

x-intercepts = 2

y-intercepts = 1

Example #4: Find the center and the radius of the circle. Sketch and identify the number of "x" and "y" intercepts.

$$x^2 + y^2 + 12x - 4y + 21 = 0$$

$$x^2 + 12x + \frac{36}{(6)^2} + y^2 - 4y + \frac{4}{(-2)^2} = -21 + 40$$


$$(x + 6)^2 + (y - 2)^2 = 19$$

Center: $(-6, 2)$

x-intercepts = 2

Radius: $\sqrt{19}$

y-intercepts = 0

more than 4 steps

$$\sqrt{16} = 4 \quad \text{less than 5 steps}$$

$$\sqrt{25} = 5$$