Math 3 Unit 3: Polynomial Functions

Unit	Title	Standards
3.1	End Behavior of Polynomial Functions	F.IF.7c
3.2	Graphing Polynomial Functions	F.IF.7c, A.APR3
3.3	Writing Equations of Polynomial Functions	F.IF.7c
3.4	Factoring and Graphing Polynomial Functions	F.IF.7c, F.IF.8a, A.APR3
3.5	Factoring By Grouping	F.IF.7c, F.IF.8a, A.APR3
3.6	More Factoring and Graphing Polynomial Functions	F.IF.7c, F.IF.8a, A.APR3
3.7	Factoring By Division	F.IF.7c, F.IF.8a, A.APR2, A.APR3, A.APR.6
Unit 3 Review		

Additional Clovis Unified Resources

http://mathhelp.cusd.com/courses/math-3

Clovis Unified is dedicated to helping you be successful in Math 3. On the website above you will find videos from Clovis Unified teachers on lessons, homework, and reviews. Digital copies of the worksheets, as well as hyperlinks to the videos listed on the back are also available at this site.

Math 3 Unit 3: Online Resources

3.1	End Behavior of Polynomial Functions	• Khan Academy: Intro to End Behavior of Polynomials http://bit.ly/31ebb • Khan Academy: End Behavior of Functions & their Graphs http://bit.ly/31ebc • Khan Academy: Article - End Behavior of Polynomials http://bit.ly/31ebd • Purple Math: End Behavior http://bit.ly/31ebp • Yay Math: Polynomial Functions http://bit.ly/31ebg
3.2	Graphing Polynomial Functions	• Math is Fun: Solving Polynomials http://bit.ly/32gpfa • Purple Math: Degrees, Turnings, and "Bumps" http://bit.ly/32gpfb • Purple Math: Zeroes and their Multiplicities http://bit.ly/32gpfc • Purple Math: Quickie Graphing of Polynomials http://bit.ly/32gpfe • Khan Academy: Zeros of Polynomials & their Graphs http://bit.ly/32gpff • Khan Academy: Article - Graphs of Polynomials http://bit.ly/32gpfg • Graphing a Polynomial Function in Factored Form http://bit.ly/32gpfh and http://bit.ly/32gpfi
3.3	Writing Equations of Polynomial Functions	• How to Write the Equation of a Polynomial Function from its Graph http://bit.ly/33wepfa • Purple Math: More about Zeros and their Multiplicities (see example 3) http://bit.ly/33wepfb
3.4	Factoring & Graphing Polynomials	• She Loves Math: Graphing and Finding Roots of Polynomial Functions http://bit.ly/34fgpb • Graphing Polynomial Functions by Factoring http://bit.ly/34fgpc
3.5	Factoring By Grouping	• Khan Academy: Factoring and Graphing Polynomial Equations by Grouping http://bit.ly/35fga • Khan Academy: Zeros of Polynomials & their Graphs http://bit.ly/35fgb
3.6	More Factoring and Graphing Polynomial Functions	• Graphing Polynomial Functions by Factoring http://bit.ly/34fgpc • Khan Academy: Factoring and Graphing Polynomial Equations by Grouping http://bit.ly/35fga • Khan Academy: Zeros of Polynomials & their Graphs http://bit.ly/35fgb
3.7	Factoring By Division	• Zeros of a Polynomial Function by Factoring, Graphing, and Synthetic Division http://bit.ly/36fda • Patrick JMT: Long Division of Polynomials http://bit.ly/36fdb • Cool Math: Long Division with Polynomials http://bit.ly/36fdd • Patrick JMT: Finding all the Zeros of a Polynomial – Synthetic Division http://bit.ly/36fdi and http://bit.ly/36fdj

Math 3 Unit 3 Worksheet 1 **Name:**_____
End Behavior of Polynomial Functions **Date:**_____ **Per:**_____

Identify the leading coefficient, degree, and end behavior.

1. $f(x) = 5x^2 + 7x - 3$
   Degree:
   Leading Coeff:
   End Behavior:

2. $y = -2x^2 - 3x + 4$
   Degree:
   Leading Coeff:
   End Behavior:

3. $g(x) = x^3 - 9x^2 + 2x + 6$
   Degree:
   Leading Coeff:
   End Behavior:

4. $y = -7x^3 + 3x^2 + 12x - 1$
   Degree:
   Leading Coeff:
   End Behavior:

5. $h(x) = -2x^7 + 5x^4 - 3x$
   Degree:
   Leading Coeff:
   End Behavior:

6. $g(x) = 8x^3 + 4x^2 + 7x^4 - 9x$
   Degree:
   Leading Coeff:
   End Behavior:

Identify the end behavior. Justify your answer.

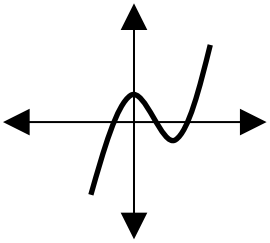
7. $f(x) = 4x^5 - 3x^4 + 2x^3$

8. $y = -x^4 + x^3 - x^2 + 1 - 1$

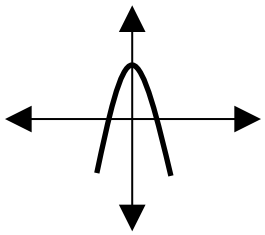
9. $h(x) = 3x^6 - 7x^4 - 2x^9$

Identify whether the function graphed has an odd or even degree and a positive or negative leading coefficient. Justify your answer.

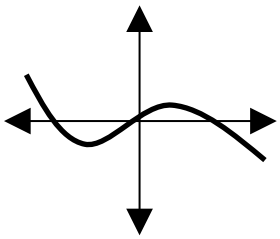
10.
deg:
coeff:
justify:



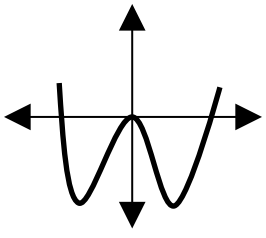
11.
deg:
coeff:
justify:



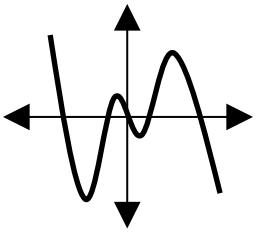
12.
deg:
coeff:
justify:



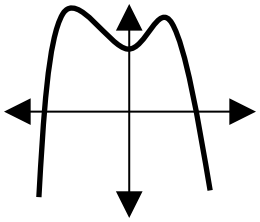
13.
deg:
coeff:
justify:



14.
deg:
coeff:
justify:



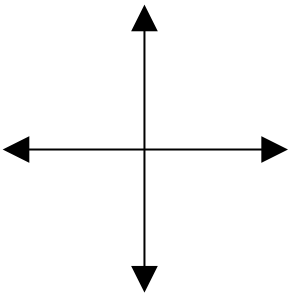
15.
deg:
coeff:
justify:



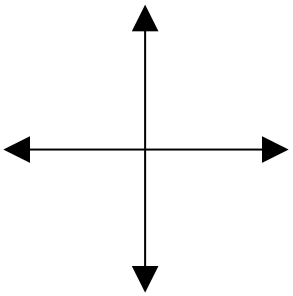
16. Write a polynomial function with end behavior of:
on the left $f(x)$ goes to + ∞ and on the right $f(x)$ goes to $-\infty$.

17. Write a polynomial function with end behavior of:
on the left $f(x)$ goes to + ∞ and on the right $f(x)$ goes to $+\infty$.

18. Sketch a graph of a polynomial function with a negative lead coefficient and an even degree.



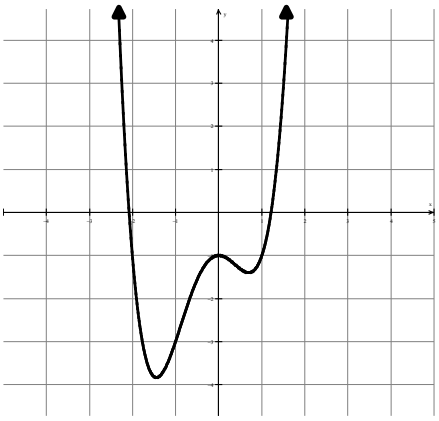
19. Sketch a graph of a polynomial function with a positive lead coefficient and an odd degree.



20. The equation of the polynomial function to the right is

$$f(x) = x^4 + x^3 - 2x^2 - 1$$

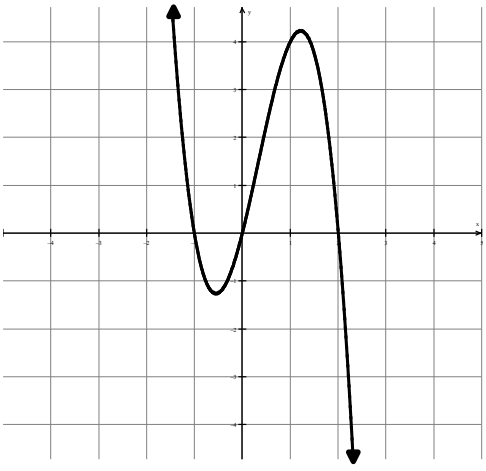
Write an equation for a translation of $f(x)$ that has no x-intercepts.
(If not possible, explain why.)



21. The equation of the polynomial function to the right is

$$g(x) = -2x^3 + 2x^2 + 4x$$

Write an equation for a translation of $g(x)$ that has no x-intercepts.
(If not possible, explain why.)



Determine the degree of the polynomial in factored form. Then demonstrate that you are correct by writing the polynomial in standard form.

22. $y = (x + 3)(x^2 - 5x - 4)$

23. $y = x^3(x - 2)^2(x + 1)$

24. $y = x(x + 3)(x - 1)^2$

25. Describe in words how you can know the degree without multiplying out to write the polynomial in standard form.

Math 3 Unit 3 Worksheet 2 **Name:**_____
Graphing Polynomial Functions **Date:**_____ **Per:**_____

For the functions below, identify each of the listed characteristics.

1. $y = (x-2)(x+5)(x-1)$

a) degree & leading coefficient

b) end behavior

c) x-intercepts with multiplicity

d) y-intercept

e) How many distinct x-intercepts?

f) How many roots are there?

2. $f(x) = x^2(x+2)(x-7)$

a) degree & leading coefficient

b) end behavior

c) x-intercepts with multiplicity

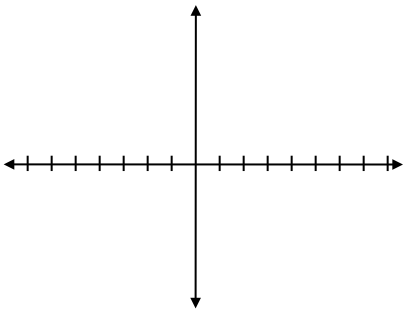
d) y-intercept

e) How many distinct x-intercepts?

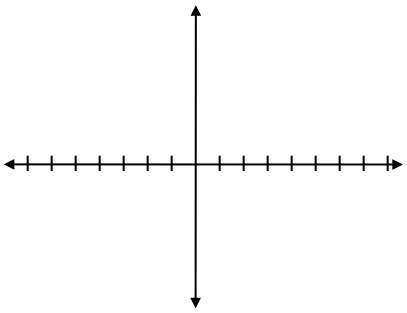
f) How many zeros are there?

Sketch graphs of the polynomial functions. Label all x and y intercepts.

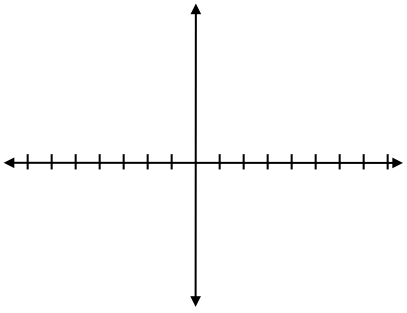
3. $y = (x-1)(x+3)(x-4)$



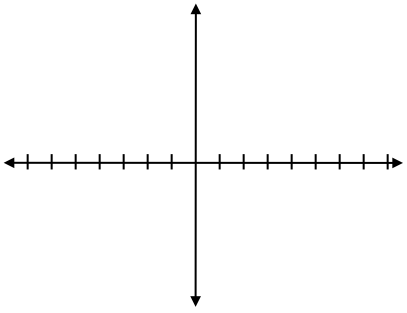
4. $f(x) = 2(x+4)(x^2+2x-8)$



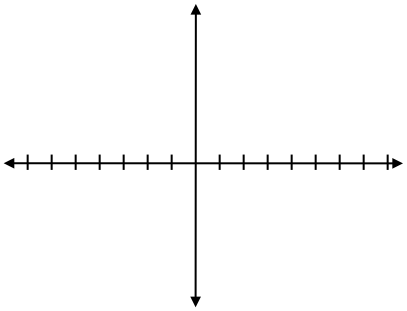
5. $g(x) = -x^2(x-3)$



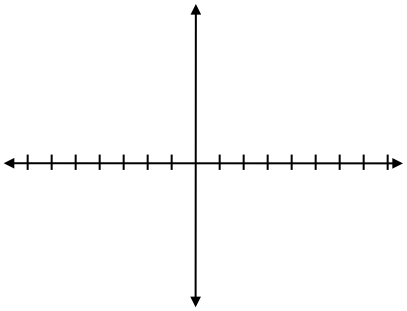
6. $f(x) = -\frac{1}{2}(x+1)(x+2)(x-4)$



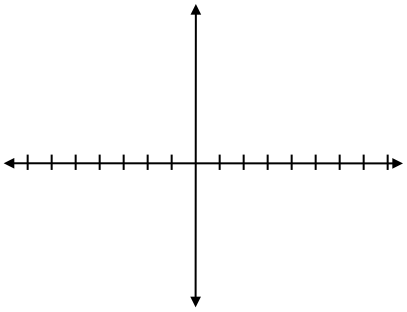
7. $y = x^2(x^2 + 2x - 15)$



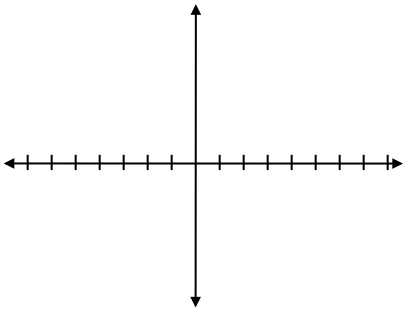
8. $h(x) = -\frac{1}{9}(x + 2)(x - 3)^2(x - 1)$



9. $f(x) = 2(x + 4)(x + 2)(2 - x)(x - 1)$

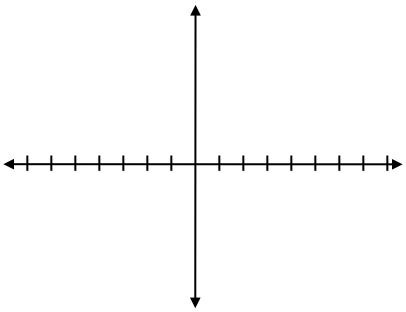


10. $g(x) = -5x^2(x + 3)^2$

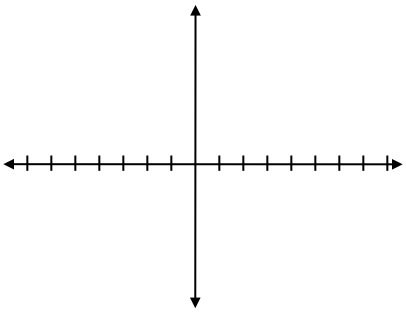


Given $f(x) = (x + 1)(x - 2)^2$. Sketch the following functions and label the indicated points.

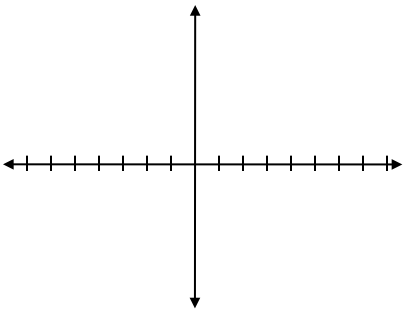
11. Sketch $y = f(x)$
x and y intercepts



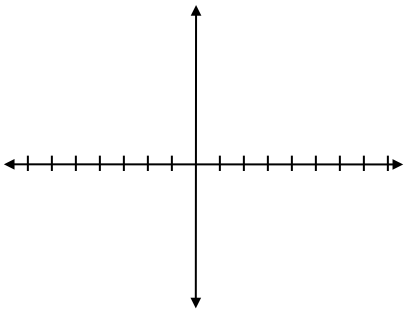
12. Sketch $y = f(x) + 3$
relative minimums and y intercept



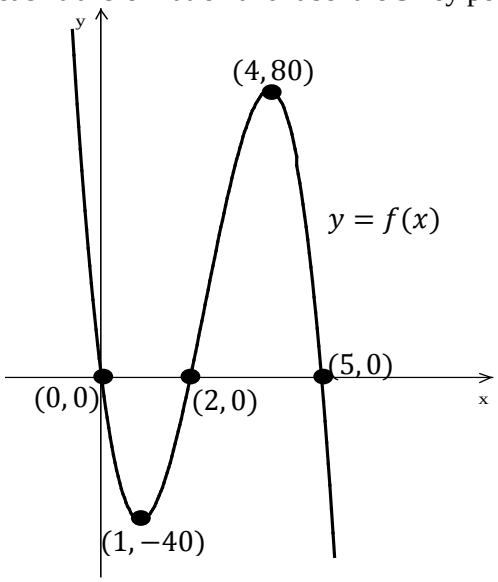
13. Sketch $y = f(x + 3)$
x intercepts and relative minimums



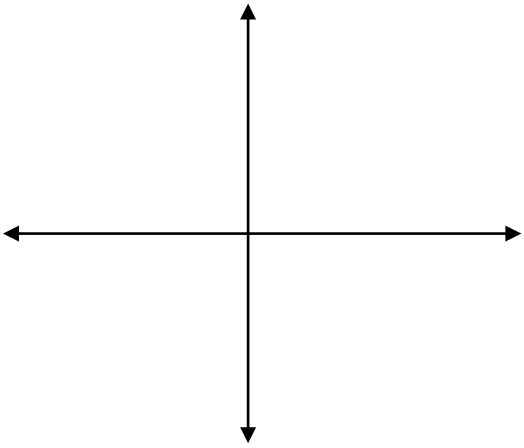
14. $y = -f(x)$
x and y intercepts



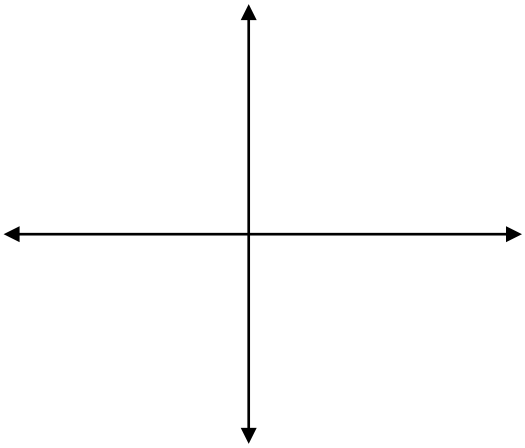
[15-18]: Given the graph of $f(x)$ with the five key labeled points. Describe the transformation in words, then sketch each transformation and label the 5 key points (x, y).



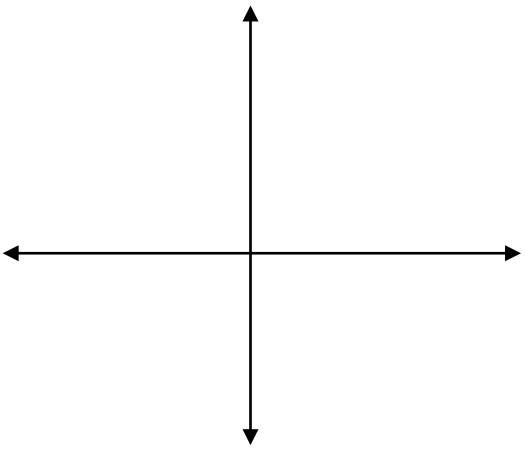
15) $f(x + 2)$
Describe the translation:



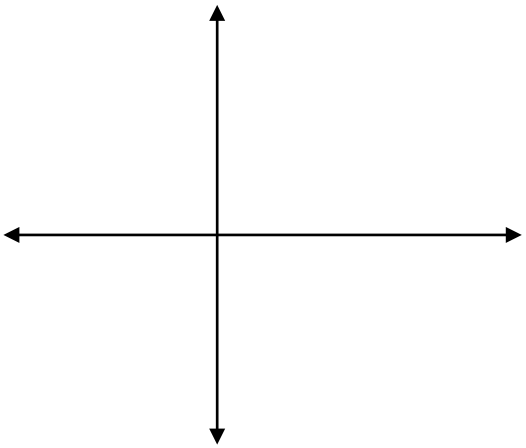
16) $f(-x)$
Describe the translation:



17) $-f(x) - 40$
Describe the translation:



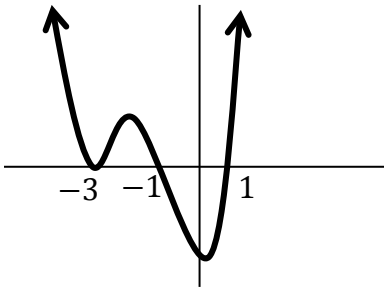
18) $f(x - 3) + 5$
Describe the translation:



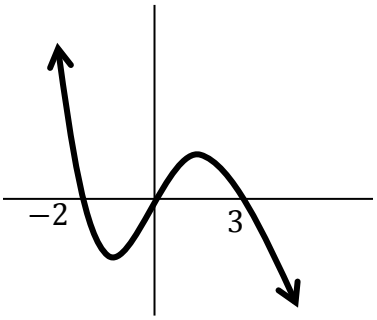
Math 3 Unit 3 Worksheet 3 **Name:**_____
Writing Equations of Polynomial Functions **Date:**_____ **Per:**_____

For #1-2, write an equation for the polynomial graph shown and determine if the leading coefficient, a, is $+$ or $-$.

1.

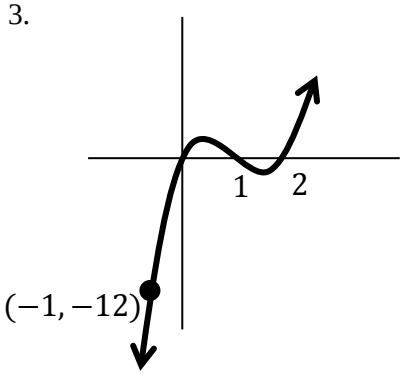


2.

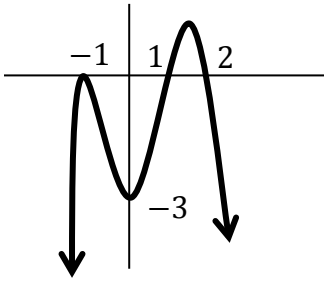


For #3-4, write the equation for the polynomial graph shown with the lowest possible degree.

3.

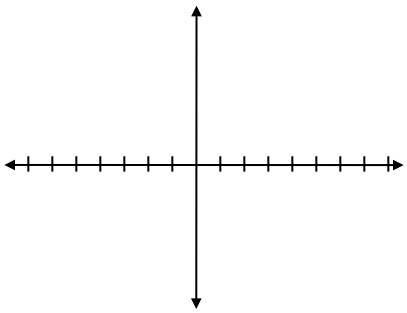


4.

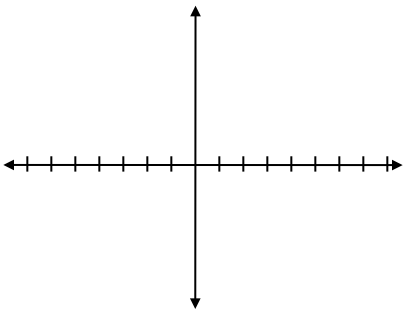


Sketch the polynomial function containing the given points and write the equation.

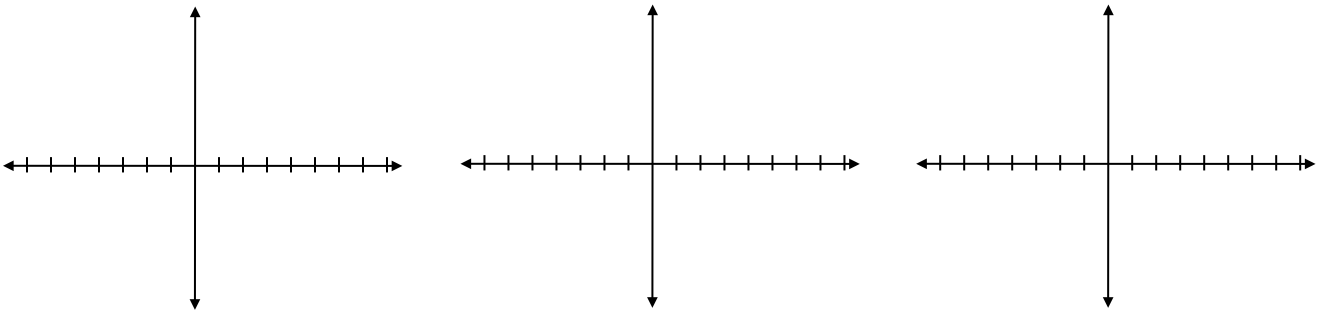
5. Cubic
$(-7, 0); (-4, 0); (0, 7); (3, 0)$



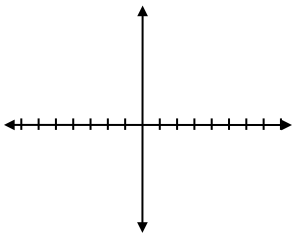
6. Quartic
$(-6, 0); (-2, 0); (0, -20); (4, 0); (5, 0)$



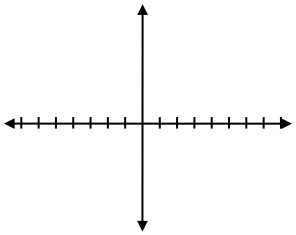
7. Sketch three different quartic functions which contain the given points: $(-7, 0); (-4, 0); (0, 5); (3, 0)$



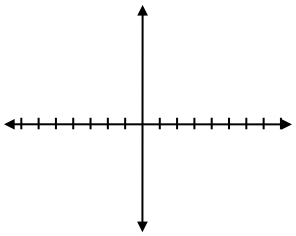
8. Sketch a graph and write the equation of a quartic function whose graph passes through $(0, -2)$ and is tangent to the x-axis at $(-1, 0)$ and $(2, 0)$.



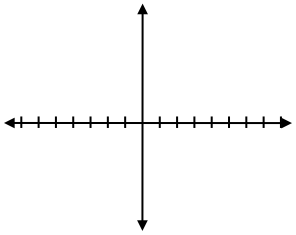
9. Sketch and write an equation of a polynomial that has the following characteristics: crosses the x-axis only at -2 and 4, touches the x-axis at 0 and 2, and is above the x-axis between 2 and 4. Determine if the leading coefficient, a, is $+$ or $-$.



10. $P(x)$ is a cubic polynomial such that $P(-3) = P(-1) = P(2) = 0$ and $P(0) = -6$. Sketch $P(x)$ and write its function.



11. If $P(x)$ is a cubic polynomial such that $P(0) = 0$, $P(2) = -4$, and $P(x)$ is above the x-axis only when $x > 4$. Sketch $P(x)$ and write its function.



12. Answer the following statements as true or false regarding the graph of the cubic polynomial function: $f(x) = ax^3 + bx^2 + cx + d$

a) It intersects the y-axis in one and only one point.
b) It intersects the x-axis in at most three points.
c) It intersects the x-axis at least once.
d) It contains the origin.
e) The ends of the graph will both be going in the same direction.

Math 3 Unit 3 Worksheet 4 **Name:**_____
Factoring and Graphing Polynomial Functions **Date:**_____ **Per:**_____

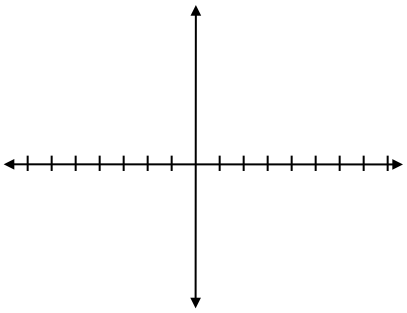
[1-6] Completely factor the following polynomials.

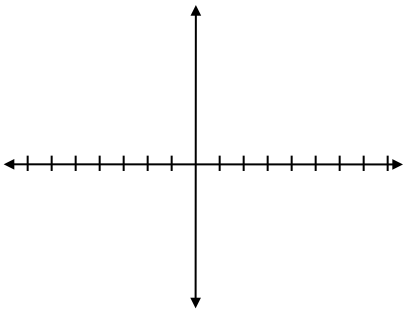
1. $x^4 - 7x^2 + 10$
2. $x^4 - 15x^2 - 16$
3. $x^4 - 10x^2 + 9$
4. $x^4 - 25x^2$
5. $-2x^3 + 2x$
6. $-x^3 + 9x$

[7-10] Factor, then sketch graphs of the polynomial functions. Label all x- and y-intercepts.

7. $y = x^3 - 16x$

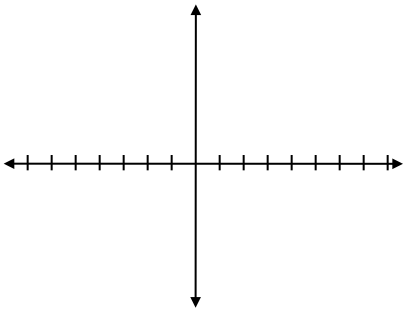
8. $y = (f \cdot g)(x)$ if $f(x) = x$ and $g(x) = -3x^2 + 12$

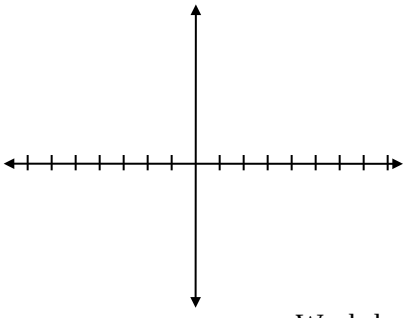




9. $y = x^4 - 10x^2 + 9$

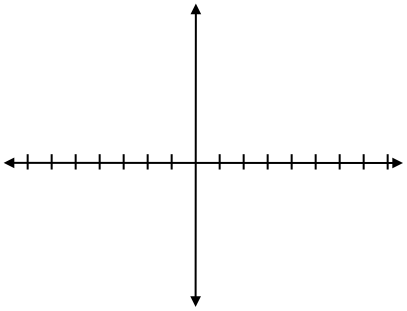
10. $y = (f \cdot g)(x)$ if $f(x) = 2x$ and $g(x) = 2x^3 - 50x$





[11-14] Factor each polynomial function, then sketch (labeling x and y intercepts) and answer the questions.

11. $y = (f - g)(x)$ if
$f(x) = 2x^3$ and $g(x) = 12x^2 - 18x$

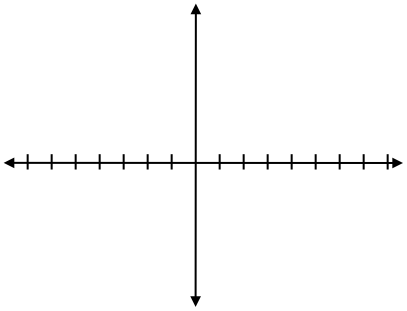


a) How many x-intercepts?

b) How many relative maximums?

c) How many relative minimums?

12. $h(x) = -x^4 - 4x^3 - 4x^2$

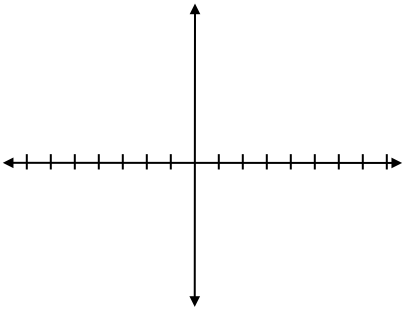


a) How many x-intercepts?

b) How many relative maximums?

c) How many relative minimums?

13. $f(x) = -x^3 + 3x^2 + 4x$

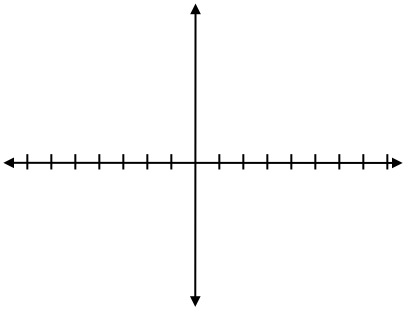


a) How many x-intercepts?

b) How many relative maximums?

c) How many relative minimums?

14. $g(x) = -5x^4 - 10x^3 + 75x^2$



a) How many x-intercepts?

b) How many relative maximums?

c) How many relative minimums?

Math 3 Unit 3 Worksheet 5 **Name:**_____
Factoring by Grouping **Date:** _____ **Per:**_____

Factor completely over the integers:

1. $x(x+2)+3(x+2)$

2. $x(x^2+5)-9(x^2+5)$

3. $x^2(x-7)-4(x-7)$

4. $x(x^2+3)-(3+x^2)$

5. $3x(x-2)-(2-x)$

6. $a(r+s)-b(r+s)+c(r+s)$

7. $a(y-1)-b(1-y)$

8. $9(2c-d)+x^2(d-2c)$

9. $ac+bc+ad+bd$

10. $5x-5+x^2-x^3$

11. $2x^3-10x^2-2x+10$

12. $3w^3-3w^2z+18w^2-18wz$

13. $7c^3-28c^2+12-3c$

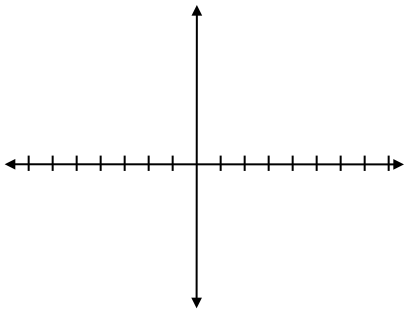
14. $12-28x-3x^2+7x^3$

15. $4x^5-x^3+8x^2-2$

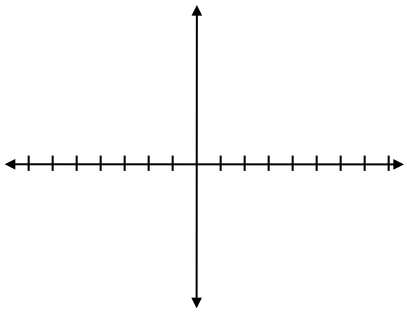
16. $k^3x+k^2x^3+k+x^2$

Factor and sketch the polynomial functions. Label x and y intercepts.

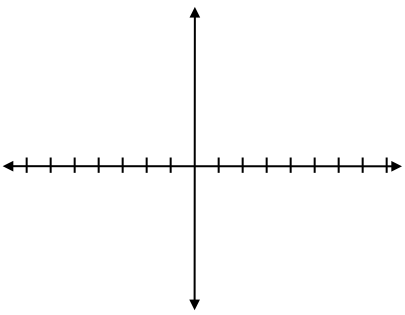
17. $y = x^3 + 4x^2 - 4x - 16$



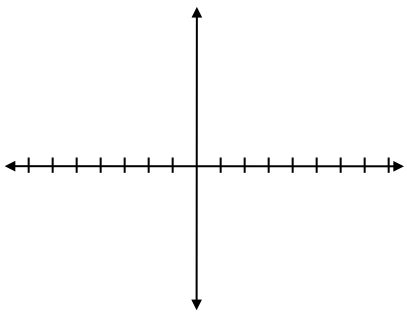
18. $f(x) = 3x^3 + 9x^2 - 27x - 81$



19. $y = (f - g)(x)$ if $f(x) = x^4 + x^3$ and $g(x) = 16x^2 + 16x$



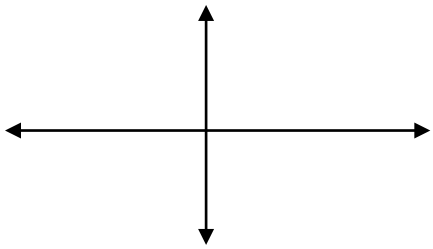
20. $y = f(x) - g(x)$ if $f(x) = 4x^4 + 8x^3$ and $g(x) = 16x^2 + 32x$



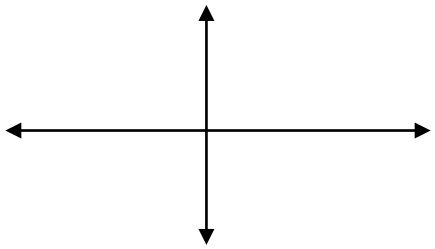
Math 3 Unit 3 Worksheet 6 **Name:**_____
More Factoring and Graphing Polynomial Functions **Date:**_____ **Per:**_____

[1-7]: Factor each function completely over the integers, find *all* of its roots (zeros), & sketch.
(Remember to label intercepts.)

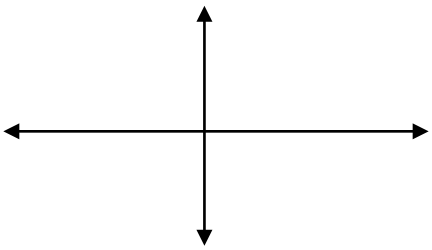
1. $f(x) = 3x^3 - 7x^2 + 12x - 28$



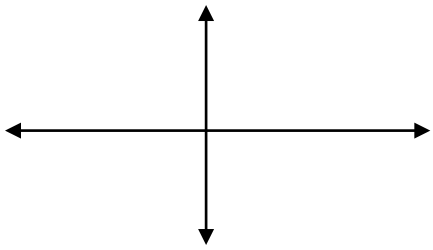
2. $g(x) = -x^3 - 9x + 4x^2 + 36$



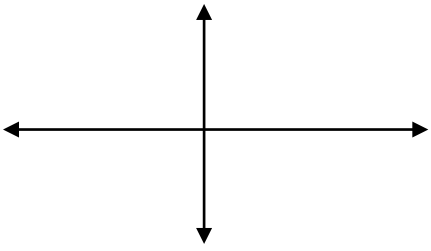
3. $h(x) = 2x^3 - x^2 - 10x + 5$



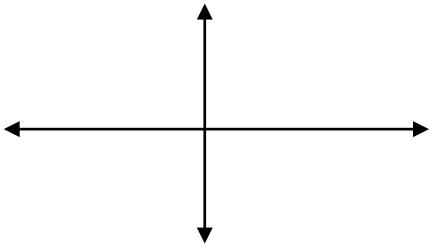
4. $f(x) = 3x^3 + 2x^2 - 36x - 24$



5. $f(x) = (g \cdot h)(x)$, given
$g(x) = -3x^2,\ h(x) = 2x^2 - 7x - 4$

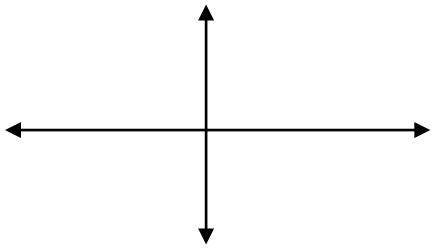


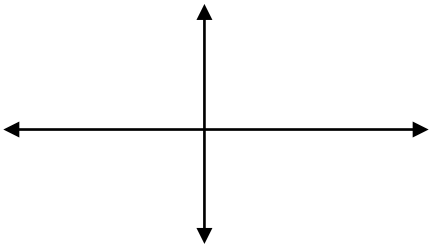
6. $g(x) = (f \cdot h)(x)$, given
$f(x) = x^2 + x - 12,\ h(x) = 3x^2 - 8x - 3$



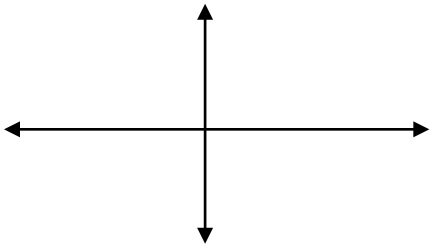
7. $h(x) = (f - g)(x)$, given
$f(x) = 2x(x + 6),\ g(x) = x^2 + 7x + 24$

8. $f(x) = x^3 - 8$





9. $g(x) = 2x^4 + 54x$



[10-12]: Write the equation for each polynomial function with lowest degree, given the following information.

10. $Roots = \{-3, 0\ mult.\ 2, 4\}$, and $f(1) = 36$

11. The graph touches the *x*-axis at $x = -4\ \&\ x = 3$, and it goes thru at $x = -1$; the *y*-intercept is -9.

12. $Zeros = \{\pm 4, -1\ mult.\ 2\}$, and the point $(2, -120)$ is on the curve.

Math 3 Unit 3 Worksheet 7 **Name:**_____
Factoring by Division **Date:**_____ **Per:**_____

Divide using polynomial long division:

1. $(x^2 + 5x - 14) \div (x - 2)$

Is $(x - 2)$ a factor of $(x^2 + 5x - 14)$?

2. $(x^2 - 2x - 48) \div (x + 5)$

Is $(x + 5)$ a factor of $(x^2 - 2x - 48)$?

3. $(x^2 + 7x + 12) \div (x + 4)$

Is $(x + 4)$ a factor of $(x^2 + 7x + 12)$?

4. $(x^3 - 3x^2 + 8x - 6) \div (x - 1)$

Is $(x - 1)$ a factor of $(x^3 - 3x^2 + 8x - 6)$?

Divide using synthetic division:

5. $(x^2 + x + 30) \div (x + 3)$

Is $(x + 3)$ a factor of $(x^2 + x + 30)$?

6. $(x^2 - 2x - 10) \div (x + 5)$

Is $(x + 5)$ a factor of $(x^2 - 2x - 10)$?

7. $(x^3 - 3x^2 - 16x - 12) \div (x - 6)$

Is $(x - 6)$ a factor of $(x^3 - 3x^2 - 16x - 12)$?

8. $(x^4 - 7x^2 - 18) \div (x + 3)$

Is $(x + 3)$ a factor of $(x^4 - 7x^2 - 18)$?

A polynomial $f(x)$ and a factor of $f(x)$ are given. Factor $f(x)$ completely.

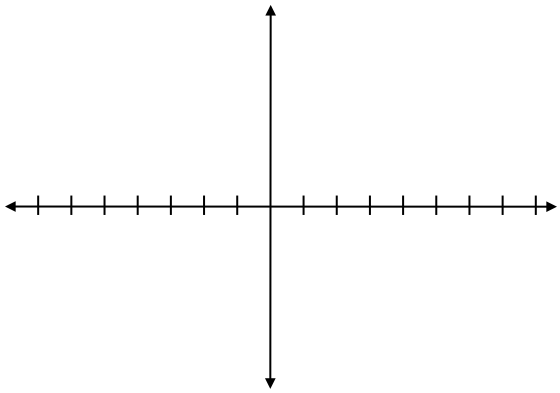
9. $f(x) = x^3 - 12x^2 + 12x + 80\,;\, x - 10$

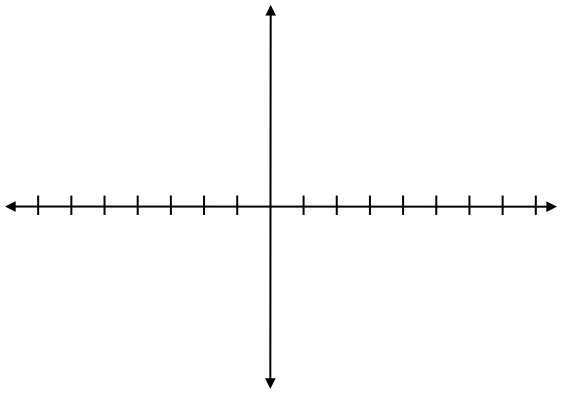
10. $f(x) = x^3 - x^2 - 21x + 45\,;\, x + 5$

Given one zero, factor and sketch the polynomial functions. Label x and y intercepts.

11. $y = 2x^3 + 3x^2 - 39x - 20\,;\; 4$

12. $y = x^3 - x^2 - 8x + 12\,;\; -3$





13. a) Divide $f(x)$ by $(x-3)$ by the method of your choice: b) What was the remainder? _____

$f(x) = x^2 - 11x + 24$

c) Find the value of $f(3)$. $f(3)$ =_____

14. a) Divide $f(x)$ by $(x+1)$ using the method of your choice: b) What was the remainder? _____

$f(x) = x^3 - x^2 + 2x + 7$

c) Find the value of $f(-1)$ $f(-1)$ =_____

15. What pattern do you notice in the answers for # 13 and 14?

16. Based on your observations in problem 15, do you think $(x-1)$ will be a factor of $(x^{13} - 2x + 1)$?

Math 3 Unit 3 **Name:**_____
Review Worksheet **Date:** _____ **Per:**_____

Directions: Show valid, appropriate, & legible work.

[1-6]: For each of the following functions: (a) Determine the end behavior. (b) What is the y-intercept?

1) $f(x) = 5x^3 - 7x^2 - 2x + 1$ 2) $g(x) = -2x^4 + 9x^3 - x^2 + x - 8$ 3) $h(x) = 4x^4 + 3x^3 + 2x^2 + x$

4) $y = -10x^3 + 3x^2 - 2x - 11$ 5) $f(x) = 2(x-3)(x+2)^2(x-5)$ 6) $g(x) = \frac{1}{3}(6-x)(x+2)^3$

Use the terms "degree" and "lead coefficient" to generalize the rules for determining end behavior.

[7-8]: Looking back at #5 & #6 …

7) How many roots (zeros) does #5 have? ____ How many distinct x-intercepts does #5 have? ____

8) How many zeros (roots) does #6 have? ____ How many distinct x-intercepts does #6 have? ____

[9-16]: (a) Write each function in completely factored form (over the integers). (b) Find <u>all</u> of the roots (real and imaginary) with their multiplicity, if different from 1. (c) How many x-intercepts does the function have?

9) $f(x) = 4x^3 + 20x^2 - 9x - 45$ 10) $g(x) = x^5 - 50x^3 + 49x$

11) $y = x^3 - 4x^2 - 12x$

12) $f(x) = x^3 - 12x^2 + 41x - 42$ if $(x - 7)$ is a factor of f.

13) $g(x) = 2x^3 + 3x^2 - 89x + 120$
if one of the roots is 5.

14) $f(x) = x^4 + 9x^2 - 36$

15) $g(x) = x^3 + 7x^2 + 34x + 66$
if $(x + 3)$ is a factor of g.

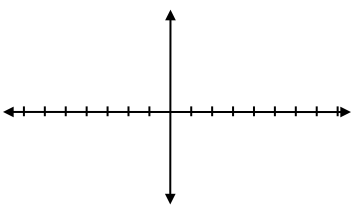
16) $y = 3x^4 - 16x^3 + 3x^2 + 46x + 24$
if two of the zeros are -1 and 4.

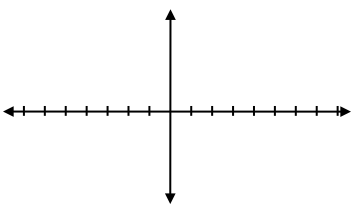
[17-21]: Sketch the graph for each of the following; label x- and y-intercepts. {Hint: Finish factoring each first.}

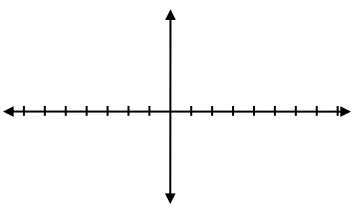
17) $y = x(x + 3)^2(2 - x)$

18) $y = x^3 - x^2 - 2x$

19) $f(x) = (x^2 - 3x - 4)(x - 4)$

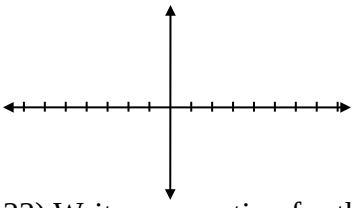


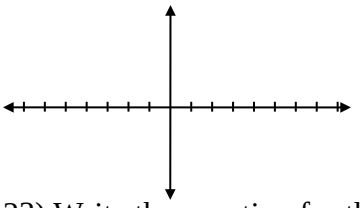




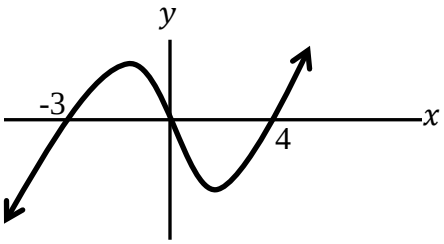
20) $g(x) = (x^2 + 2x - 3)(x^2 - 3x - 18)$

21) $h(x) = (6 - 5x - x^2)(x^2 + 3x - 4)$

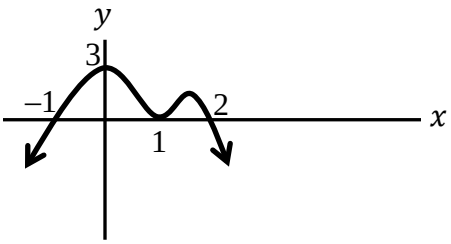




22) Write an equation for the graph below:

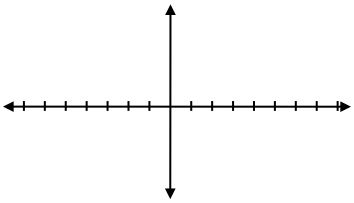


23) Write the equation for the graph below:

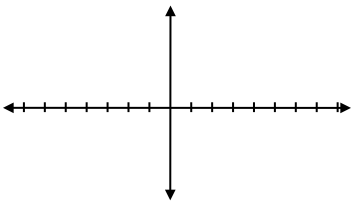


[24-27]: Sketch the graph and write, in factored form, the equation for a polynomial function, given:

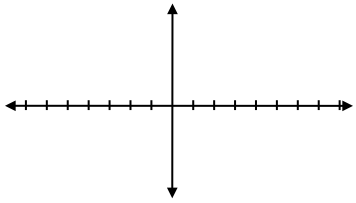
24) Quartic polynomial function with roots 0, $\frac{5}{2}$ and 3. The function crosses the *x*-axis only at $x = 0$ and $\frac{5}{2}$ and touches the *x*-axis at 3. $f(1) = 36$.



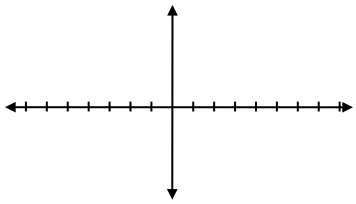
25) 4th degree polynomial whose graph passes through (0, 48) (2, 0) and (−5, 0) and is tangent to the *x*-axis at (3, 0).



26) Cubic polynomial passes through the points (1, −20), (3, 0), and (−4, 0) which has a multiplicity of 2.



27) $P(x)$ is a cubic polynomial such that $P(-1) = P(2) = P(3) = 0$ and $P(1) = -4$.



28) True/False

a) It is possible for a cubic polynomial function with real coefficients to have all imaginary roots.

b) It is possible for a 4th degree polynomial function with real coefficients to have all imaginary zeros.

c) If "r" is a root (zero) of the polynomial function $p(x)$, then $(x - r)$ is a factor of $p(x)$.

d) A polynomial function of degree "n" has "n" roots (zeros).

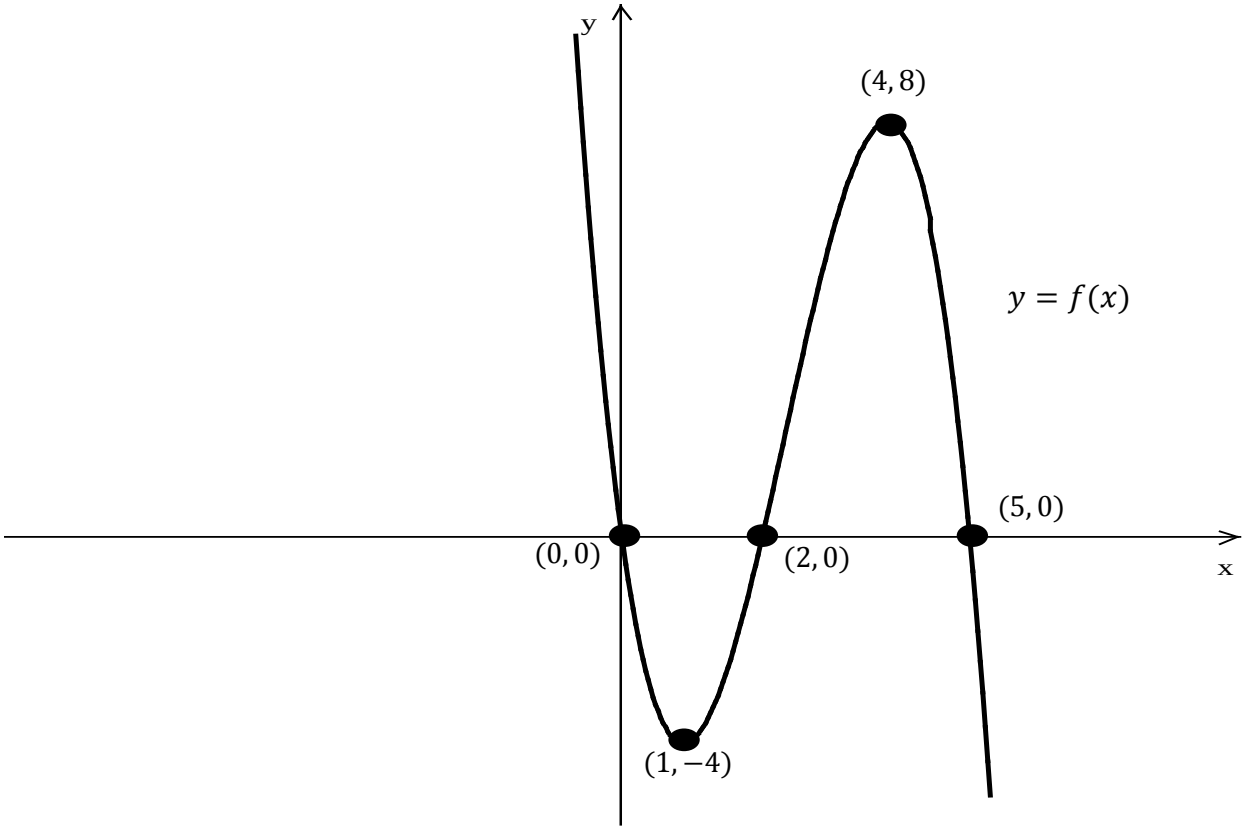
e) A polynomial function of degree "n" has "n" x-intercepts.

[29-30]: (a) Find all of the zeros (roots) for each. (b) How many x-intercepts does the function have?

29) $f(x) = x^4 + x^2 - 20$

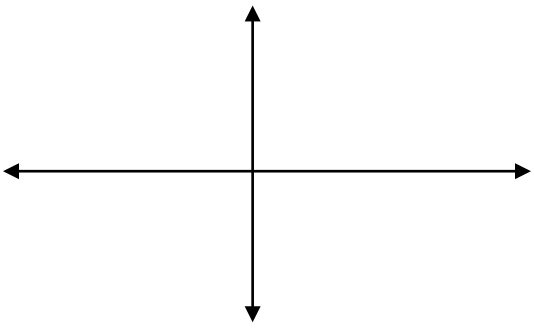
30) $g(x) = x^3 + 4x^2 - 9x - 36$

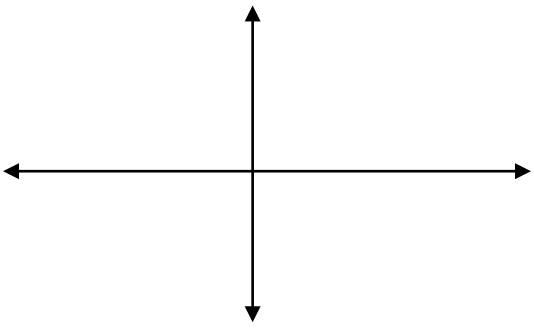
[31-38]: Given the graph of $f(x)$ with the five key labeled points. Describe the transformation in words, then sketch each transformation and label the 5 key points (x, y).



31) $f(x + 5)$
Describe the translation:

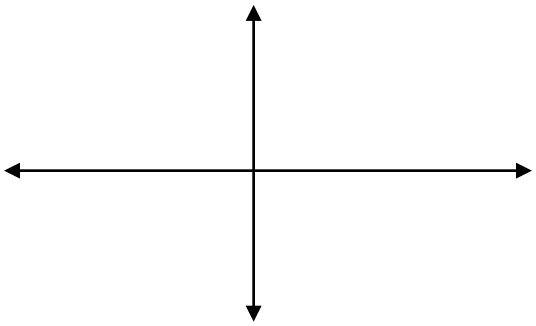
32) $f(x - 2)$
Describe the translation:



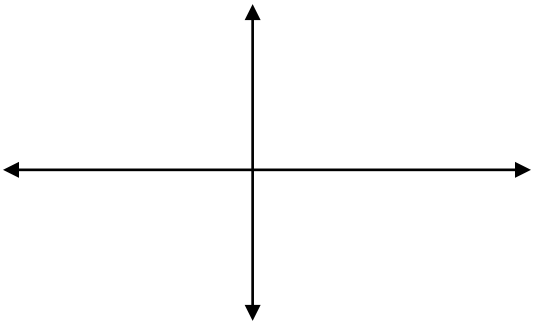


33) $f(x) + 4$
Describe the translation:

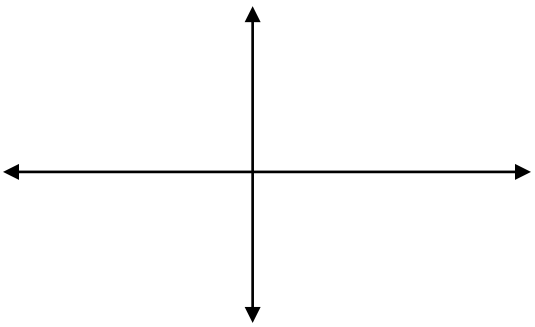
34) $f(x) - 8$
Describe the translation:



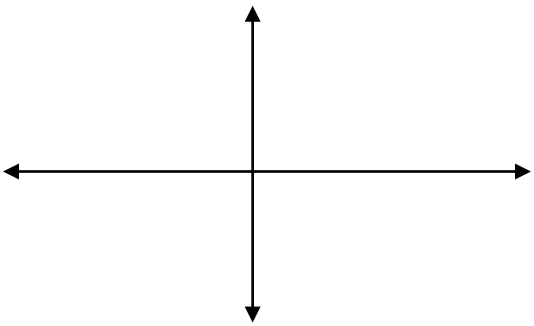
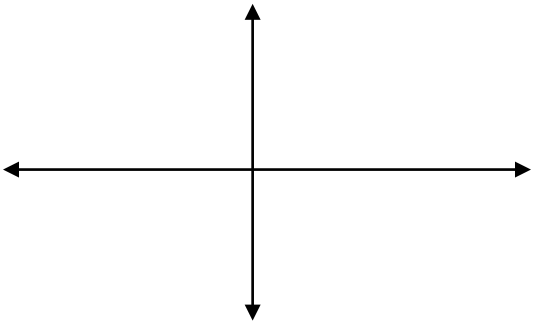
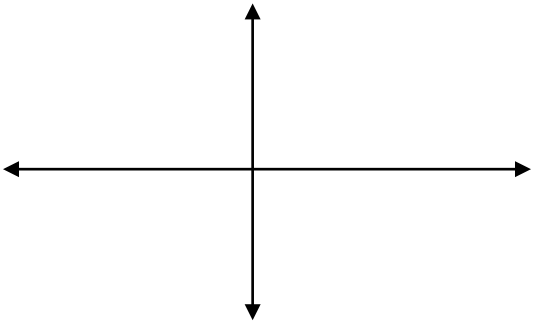
35) $-f(x) - 4$
Describe the translation:



36) $-f(x) + 8$
Describe the translation:



37) $f(x + 1) - 4$
Describe the translation:



38) $f(x - 3) + 5$
Describe the translation: