Math 3 Unit 9: Logarithms

Unit	Title	Standards
9.1	The WhatPower Function	F.LE.4.2
9.2	Introduction to Logarithms	F.LE.4.2
9.3	Solving and Evaluating Exponential & Logarithmic Equations with Common Bases	F.BF.4a F.LE.4
9.4	Graphing Logarithmic Functions	F.IF.7.e
Activity	Logarithm Rules Activity	F.LE.4.1, F.LE.4.3
9.5	Laws of Logarithms	F.LE.4.1, F.LE.4.3 A.SSE.3
9.6	Solving Logarithmic Equations using Laws of Logarithms	F.BF.4a, F.LE.4 A.SSE.3
9.7	Solving Exponential Equations without Common Bases	F.LE.4, F.LE.4.2 F.IF.8
9.8	Applications of Logarithms	A.SSE.1b, A.SSE.3c, F.IF.8b, F.IF.6
Unit 9 Review		

Math 3 Unit 9: Online Resources

9.1	The WhatPower Function	• Khan Academy: Intro to Logarithms (WhatPower) http://bit.ly/91twpfa
9.2	Introduction to Logarithms	• Purple Math: Logarithms: Introduction to "The Relationship" http://bit.ly/92itlaa • Patrick JMT: Logarithms: Properties of Logarithms – Part 1 http://bit.ly/92itlbb • Khan Academy: Evaluating Logarithms (Advanced) http://bit.ly/92itlcc
9.3	Solving and Evaluating Exponential & Logarithmic Equations with Common Bases	• Purple Math: Solving Logarithmic Equations: Solving with Exponentials http://bit.ly/93elecba • Patrick JMT: Solving Exponential Equations - Some Basic Examples http://bit.ly/93elecbb • Patrick JMT: Properties of Logarithms - Everything You Need to Know! http://bit.ly/93elecbc • Khan Academy: Logarithmic Equations: Variable in the Base http://bit.ly/93elecbd • Khan Academy: Solving Exponential Equations using Exponent Properties http://bit.ly/93elecbe and http://bit.ly/93elecbf
9.4	Graphing Logarithmic Functions	• Khan Academy: Graphs of Logarithms & Exponentials http://bit.ly/94glfaa • Khan Academy: Graphing Basic Logarithmic Functions http://bit.ly/94glfbb • Khan Academy: Graphs of Logarithmic Functions http://bit.ly/94glfcc • Purple Math: Graphing Logarithmic Functions: Intro http://bit.ly/94glfdd
9.5	Laws of Logarithms	• Patrick JMT: Properties of Logarithms - Everything You Need to Know! http://bit.ly/93elecbc • Purple Math: Basic Log Rules & Expanding Log Expressions (Pages 1-3) http://bit.ly/95lolaa • Khan Academy: Intro to Logarithm Properties (1 and 2) http://bit.ly/95lolbb and http://bit.ly/95lolcc • Khan Academy: Using the Logarithmic Product Rule (Expand) http://bit.ly/95loldd • Khan Academy: Using the Logarithmic Power Rule http://bit.ly/95lolee • Khan Academy: Using the Properties of Logarithms: Multiple Steps http://bit.ly/95lolff
9.6	Solving Logarithmic Equations using Laws of Logarithms	• Khan Academy: Logarithmic equations: Variable in the Argument http://bit.ly/96slelola • Purple Math: Solving Logarithmic Equations: Solving with Exponentials http://bit.ly/93elecba • Patrick JMT: Solving Logarithmic Equations http://bit.ly/96slelolb and http://bit.ly/96slelolc
9.7	Solving Exponential Equations without Common Bases	• Purple Math: Solving Exponential Equations with Logarithms http://bit.ly/97seeaa • Patrick JMT: Change of Base Formula for Logarithms http://bit.ly/97seebb • Patrick JMT: Solving Exponential Equations (Example a & b – up to 3:30) http://bit.ly/97seecc
9.8	Applications of Logarithms	• Khan Academy: Compound interest introduction http://bit.ly/98seeaa

Evaluate each expression using the "WhatPower" function. The first three have been completed for you.

1. $\text{WhatPower}_5(25) = 2$

2. $\text{WhatPower}_3(27) = 3$

3. $\text{WhatPower}_2(16) = 4$

4. $\text{WhatPower}_2(32) =$

5. $\text{WhatPower}_{10}(100) =$

6. $\text{WhatPower}_{10}(1000) =$

7. $\text{WhatPower}_4(64) =$

8. $\text{WhatPower}_2(64) =$

9. $\text{WhatPower}_{10}(10000) =$

10. $\text{WhatPower}_5(\sqrt{5}) =$

11. $\text{WhatPower}_9(3) =$

12. $\text{WhatPower}_8(2) =$

13. $\text{WhatPower}_{42}(1) =$

14. $\text{WhatPower}_{\frac{1}{2}}\left(\frac{1}{8}\right) =$

15. $\text{WhatPower}_{\frac{2}{3}}\left(\frac{4}{9}\right) =$

16. $\text{WhatPower}_3\left(\frac{1}{3}\right) =$

17. $\text{WhatPower}_2\left(\frac{1}{4}\right) =$

18. $\text{WhatPower}_{100}(0.01) =$

Write a definition for the function **WhatPower$_b(a)$** where b and a are numbers.

Evaluate the following, if possible, and use your definition to justify each of your results.

19. $\text{WhatPower}_7(49)$

20. $\text{WhatPower}_5(1)$

21. $\text{WhatPower}_3\left(\frac{1}{27}\right)$

22. $\text{WhatPower}_0(7)$

23. $\text{WhatPower}_1(12)$

24. $\text{WhatPower}_{-2}(16)$

25. $\text{WhatPower}_{-3}(27)$

26. $\text{WhatPower}_8(0)$

27. $\text{WhatPower}_5(-25)$

Describe the allowable values of b in the expression $\textbf{WhatPower}_b(a)$. Explain how you know.

Describe the allowable values of a in the expression $\textbf{WhatPower}_b(a)$. Explain how you know.

Math 3 Unit 9 Worksheet 2
Introduction to Logarithms

Name:_____
Date:_____Per:_____

[1-20] Evaluate the following logarithms. If needed, write an equivalent exponential equation. Do these without the use of your calculator.

1. $\log_5 25$

2. $\log_3 81$

3. $\log_4 0.25$

4. $\log_7 \left(\frac{1}{49}\right)$

5. $\log_2 \left(\frac{1}{8}\right)$

6. $\log_3 \sqrt{3}$

7. $\log_5 \sqrt[3]{5}$

8. $\log_9 3$

9. $\log_8 2$

10. $\log_{25} 1$

11. $\log_{\frac{1}{2}} \frac{1}{8}$

12. $\log_9 \left(\frac{1}{3}\right)$

13. $\log 1000$

14. $\log 100{,}000$

15. $\log 10^8$

16. $\log \left(\frac{1}{10}\right)$

17. $\ln e^4$

18. $\ln 1$

19. $\ln \left(\frac{1}{e^5}\right)$

20. $\ln \sqrt[4]{e}$

[21- 25] Find which two consecutive integers the logarithm lies between.

21. $\log_2 30$ 22. $\log_7 9$ 23. $\log_4 100$ 24. $\log_3 75$ 25. $\log_{10} 7500$

[26-28] Find the *y*-intercept for the following functions.

26. $y = \log_3(x + 9) - 1$ 27. $y = \log_2(x + 16) + 3$ 28. $y = \log(x + 1) + 4$

Application

In chemistry, the pH of a solution is defined by the equation pH = – log (H) where H represents the concentration of hydrogen ions in the solution. Any solution with pH less than 7 is considered acidic and any solution with a pH greater than 7 is considered basic. Fill in the table below. Round your pH's to the nearest thousandth of a unit.

Substance	Concentration of Hydrogen	pH	Basic or Acidic?
Milk	1.6×10^{-7}		
Coffee	1.3×10^{-5}		
Bleach	2.5×10^{-13}		
Lemon Juice	7.9×10^{-2}		
Rain	1.6×10^{-6}		
Egg Whites	1×10^{-8}		

Reasoning

Can the value of $\log_2(-4)$ be found? What about the value of $\log_2 0$? Why or why not? What does this tell you about the domain of $\log_b x$?

Math 3 Unit 9 Worksheet 3
Solving and Evaluating Exponential &
Logarithmic Equations with Common Bases

Name:_____
Date:_____Per:_____

Solve for x.

1. $9^x = \frac{1}{27}$

2. $8^{2+x} = 2$

3. $4^{1-x} = 8$

4. $27^{2x-1} = 3$

5. $4^{3x+5} = 16^{x+1}$

6. $3^{-(x+5)} = 9^{4x}$

7. $25^{2x} = 5^{x+6}$

8. $6^{x+1} = 36^{x-1}$

9. $10^{x-1} = 100^{4-x}$

10. $5^x = \sqrt{125}$

11. $6^x = 36\sqrt{6}$

12. $49^{x-2} = 7\sqrt{7}$

Evaluate each logarithm.

13. $\log_6 6\sqrt{6}$

14. $\log_5 125\sqrt{5}$

15. $\log_4 \sqrt{2}$

16. $\log_{27} \sqrt{3}$

17. $\log_7 \sqrt[3]{49}$

18. $\log_3 \sqrt[5]{9}$

19. $\log_{\frac{1}{2}} 8$

20. $\log_{\frac{1}{3}} 81$

21. $\log_2 \sqrt[3]{\frac{1}{4}}$

22. $\log \frac{1}{\sqrt{1000}}$

23. $\log_8 \sqrt[3]{4}$

24. $\ln \frac{1}{\sqrt[3]{e^2}}$

Solve for x.

25. $\log_2 7x = \log_2 77$

26. $\log_6 \frac{x}{4} = \log_6 5$

27. $\log_3 2^x = \log_3 16$

28. $\log_5(2x + 12) = \log_5(3x + 4)$

29. $\log_8 3^{2x} = \log_8 81$

30. $\log 5^{4x} = \log 125$

31. $\ln 4^x = \ln 32$

32. $\log x^{\frac{5}{3}} = \log 32$

33. $\log_\pi x = 3$

34. $\log_{64} 32 = x$

35. $\log_5(2x - 7) = 0$

36. $\log(3x + 1) = 2$

37. $\ln x = 2$

38. $\log \sqrt[3]{100} = x$

39. $\ln(x - 9) = 1$
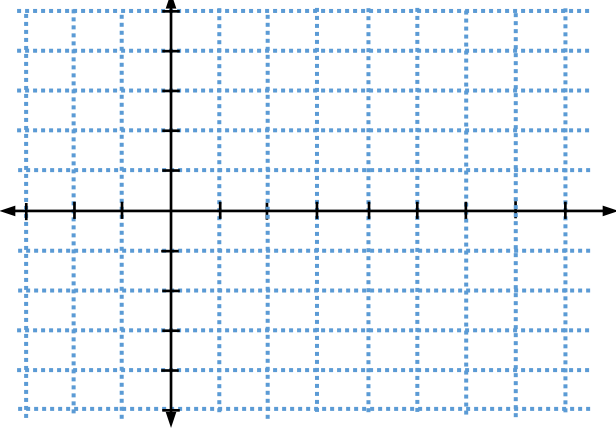
40. $\log_x 27 = \frac{3}{4}$

Math 3 Unit 9 Worksheet 4
Graphing Logarithmic Functions
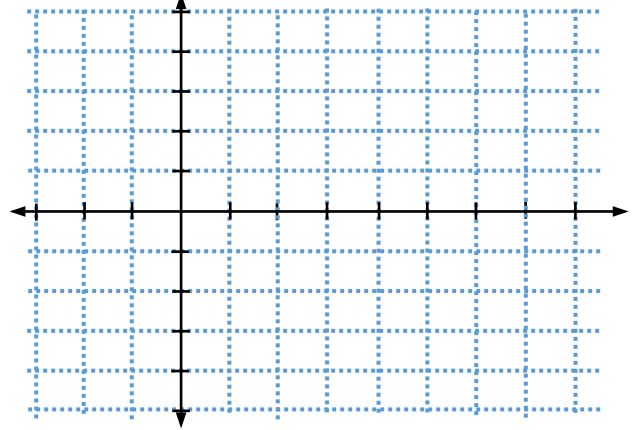
Name:_____
Date:_____ Per:_____

[1-8] For the following logarithmic functions: graph, find the x- and y-intercept(s), state the domain and range and find the equation of the vertical asymptote.
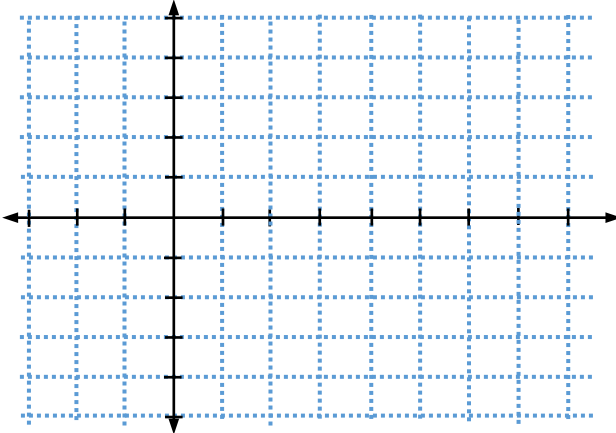
1. $y = \log_3 x$

x-intercept:_____

y-intercept:_____

Domain:_____

Range:_____

Vertical Asymptote:_____

2. $y = \ln x + 3$

x-intercept:_____

y-intercept:_____

Domain:_____

Range:_____

Vertical Asymptote:_____

3. $f(x) = \log(x + 3)$

x-intercept:_____

y-intercept:_____

Domain:_____

Range:_____

Vertical Asymptote:_____

4. $y = \log_5 x - 1$

x-intercept:_____

y-intercept:_____

Domain:_____

Range:_____

Vertical Asymptote:_____

5. $f(x) = -\ln(x - 4)$

x-intercept:_____

y-intercept:_____

Domain:_____

Range:_____

Vertical Asymptote:_____

6. $y = \log_4(x + 1) - 2$
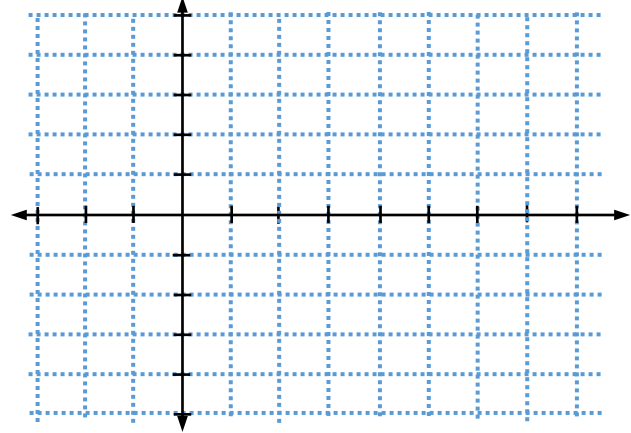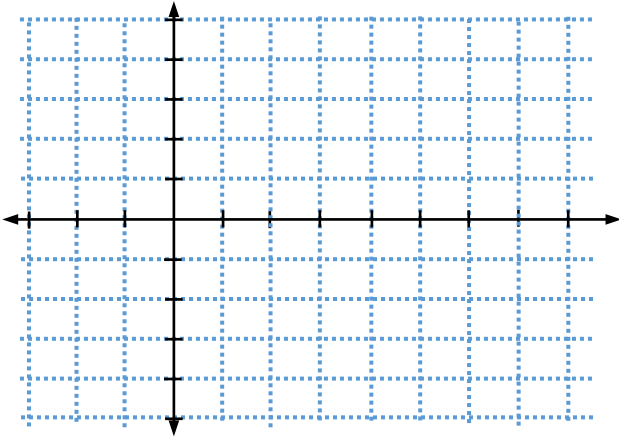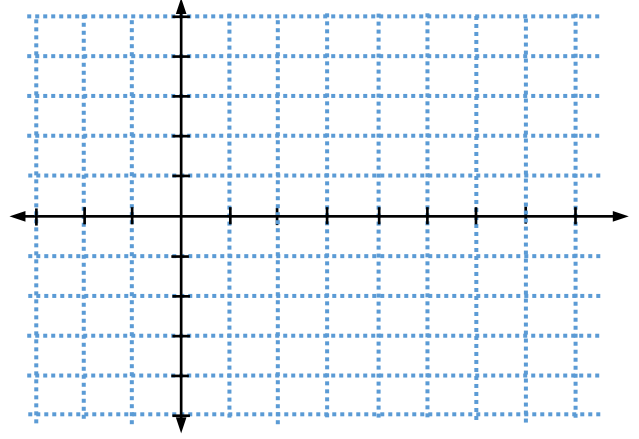
x-intercept:_____
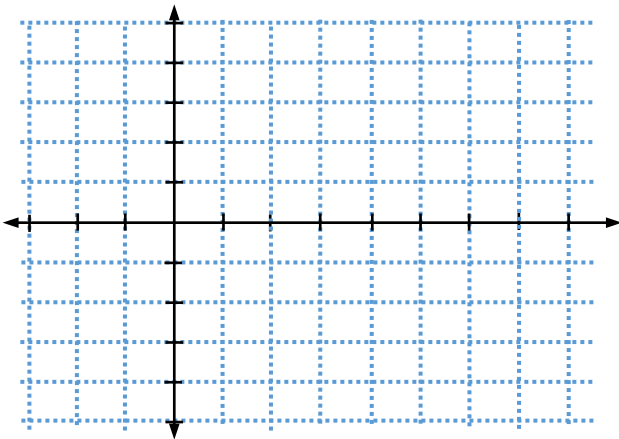
y-intercept:_____

Domain:_____

Range:_____

Vertical Asymptote:_____

7. $f(x) = 2 - \log_2(x + 1)$

x-intercept:_____

y-intercept:_____

Domain:_____
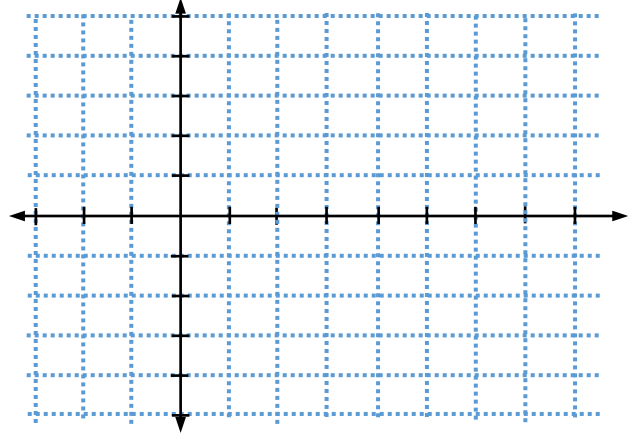
Range:_____

Vertical Asymptote:_____

Average rate of change on [1, 7]:

_____

8. $y = \log_4(x - 3) + 1$

x-intercept:_____

y-intercept:_____

Domain:_____

Range:_____

Vertical Asymptote:_____

Average rate of change on [4, 7]:

_____

Part I: Evaluate these expressions using your understanding of logs:

1) $\log_2(8) + \log_2(4) = $ _____   which is the same as $\log_2(\_\_\_)$

2) $\log_3(27) + \log_3(3) = $ _____   which is the same as $\log_3(\_\_\_)$

3) $\log_4(4) + \log_4(16) = $ _____   which is the same as $\log_4(\_\_\_)$

4) $\log_5(5) + \log_5(1) = $ _____   which is the same as $\log_5(\_\_\_)$

5) What pattern seems to hold? Write a rule:

$$\boxed{\log_b X + \log_b Y = \log_b(\_\_\_\_\_)}$$

Use the new rule to solve some problems that were impossible before:

6) $\log_6 12 + \log_6 3$ 7) $\log 250 + \log 40$ 8) $\log_8\left(\frac{3}{64}\right) + \log_8\left(\frac{1}{3}\right)$

Part II: Evaluate these expressions using your understanding of logs:

1) $\log_5(125) - \log_5(5) = $ _____   which is the same as $\log_5(\_\_\_)$

2) $\log_2(32) - \log_2(8) = $ _____   which is the same as $\log_2(\_\_\_)$

3) $\log_3(81) - \log_3(3) = $ _____   which is the same as $\log_3(\_\_\_)$

4) $\log_4(64) - \log_4(16) = $ _____   which is the same as $\log_4(\_\_\_)$

5) What pattern seems to hold? Write a rule:

$$\boxed{\log_b X - \log_b Y = \log_b(\_\_\_\_\_)}$$

Use the new rule to solve some problems that were impossible before:

6) $\log_6 72 - \log_6 2$ 7) $\log 12 - \log 0.12$ 8) $\log_{12} 2 - \log_{12} 288$

Part III: Evaluate these expressions using your understanding of logs:

1) $\log_2(4^3) = $ ____   which is the same as   ___ $\bullet \log_2 4$

2) $\log_3(9^2) = $ ____   which is the same as   ___ $\bullet \log_3 9$

3) $\log_5(25^2) = $ ____   which is the same as   ___ $\bullet \log_5 25$

4) $\log_7(7^{-2}) = $ ____   which is the same as   ___ $\bullet \log_7 7$

5) What pattern seems to hold? Write a rule:

$$\log_b(X^c) = $$

Use the new rule to solve some problems that were impossible before:

6) $\log_3(9^{250})$ 　　　　　　　　　7) $\log_n(n^y)$ 　　　　　　　　　8) $\log\sqrt{1000}$

Part IV: Combining the three rules together: Recall that

$$\log_b(XY) = \qquad\qquad \text{and} \qquad \log_b X + \log_b Y = $$

$$\log_b\left(\frac{X}{Y}\right) = \qquad\qquad \text{and} \qquad \log_b X - \log_b Y = $$

$$\log_b(X^c) = \qquad\qquad \text{and} \qquad c\bullet\log_b X = $$

1) Write three different expressions, using each rule once, that are all equivalent to log 8.

$\log 8 = \log (\_\_\bullet\_\_) = \log(\ \ ) + \log(\ \ )$ 　　　$\log 8 = \log\left(\frac{\ \ }{\ \ }\right) = \log(\ \ ) - \log(\ \ )$ 　　$\log 8 = \log(\ \ ) = \_\_\bullet\log(\ \ )$

Expand each logarithm:

2) $\log (ac)$ 　　　　3) $\ln\left(\frac{c}{b}\right)$ 　　　　　　　4) $\log\left(\frac{\sqrt{a}}{c^2}\right)$ 　　　　　　5) $\ln\left(\frac{a^4\sqrt{b}}{c^5}\right)$

Express as a single logarithm

6) $2 \log b + 3 \log c$ 　　　　　　7) $2 \log a - 4 \log b$ 　　　　　8) $\frac{1}{2} \ln a + 2 \ln c - \ln b$

9) Express as a single logarithm and evaluate: $2\log_3 6 + 4\log_3 2 - 2\log_3 8$

10) If $\log 4 = m$ and $\log 6 = n$, then evaluate: a) log 24 　　　　　b) $\log \frac{3}{2}$ 　　　　　c) log 2

Name:_____
Date:_____Period:_____

Completely expand and simplify each expression.

1. $\log_3 9x$

2. $\ln 15x$

3. $\log 10x^4$

4. $\log_5 x^5$

5. $\log_2 \frac{2}{5}$

6. $\ln \frac{e^3}{5}$

7. $\log_4 \frac{64}{5y}$

8. $\ln ex^2 \sqrt{y}$

9. $\log_2 16x^3yz^2$

10. $\log_6 36x^2$

11. $\ln e^2 \sqrt[4]{x}$

12. $\log 1000x^3\sqrt[3]{y}$

13. $\log_2 \sqrt{x}$

14. $\ln \frac{e^4 \sqrt{x}}{y^2}$

15. $\ln \sqrt[4]{x^3}$

Condense each expression into a single logarithm and simplify if possible.

16. $\log_4 7 - \log_4 10$

17. $\ln 12 - \ln 4$

18. $2\log x + \log 11$

Condense each expression into a single logarithm and simplify if possible.

19. $6 \ln x + 4 \ln y$

20. $5 \log x - 3 \log 2$

21. $5 \log_4 2 + 7 \log_4 x + 4 \log_4 y$

22. $\ln 40 + 2 \ln \frac{1}{2} + \ln x$

23. $2 \log_5 4 + \frac{1}{3} \log_5 x$

24. $6 \ln 2 - 4 \ln y$

25. $2 \log_6 2 + 2 \log_6 3$

26. $\log 5 + 2 \log 4 - 3 \log 2$

27. $2 \log_4 3 + \frac{1}{2} \log_4 36 - 3 \log_4 3$

{hint: For #28-30, rewrite the constant as a log}

28. $1 + \log_3 x - 2 \log_3 y$

29. $3 - \frac{1}{2} \log_2 x + \log_2 y$

30. $2 - \ln x - 3 \ln y^2$

Math 3 Unit 9   Worksheet 5

Solve for x.

1. $3 \log_5 4 = \log_5 2x$

2. $2 \ln 9 = \ln 3x$

3. $2 \log_8 x = \log_8 100$

4. $\log_7 x = \log_7 2 + \log_7 3$

5. $\log_6 x = 2 \log_6 3 + \log_6 5$

6. $\log_5(x + 3) = \log_5 8 - \log_5 2$

7. $\log x - \log(x - 5) = \log 6$

8. $\ln(3x + 5) - \ln(x - 5) = \ln 8$

9. $\log_{11} x = \frac{3}{2} \log_{11} 9 + \log_{11} 2$

10. $\log x^{\frac{4}{3}} = \log 32 - \log 2$

11. $\log_6 9 + \log_6 x = 2$

12. $\log x + \log 25 = 3$

13. $\log_2 52 - \log_2 x = 2$

14. $2\log_6 2 + \log_6 18x = 3$

15. $\log_\pi 5 + \log_\pi x = 7$

16. $\log_6 x + \log_6(x - 5) = 2$

17. $2\log_4 x = 3$

18. $\ln x + \ln 5 = 4$

19. $\ln x - \ln 6 = 2$

20. $\log_2 4x - \log_2(x - 1) = 3$

21. $\log_2 x + \log_2(x - 6) = 4$

22. $2 \log 2 + \log x = 2$

23. $2 \ln 7 + \ln x = 4$

24. $\log 20 + \log 5 = x$

25. $\log_6 9 + \log_6 4 = x$

26. $\log_5(2x - 7) = 0$

27. $\ln(x - 9) = 1$

28. $\log_x 2 + \log_x 4 = \frac{3}{2}$

29. Identify which step has the error in the solution of

$$2 \log_7 x = \log_7 2 + \log_7 50$$

Step 1: $2 \log_7 x = \log_7 2 \cdot 50$

Step 2: $2 \log_7 x = \log_7 100$

Step 3: $\log_7 x = \log_7 \frac{100}{2}$

Step 4: $\log_7 x = \log_7 50$

Step 5: $x = 50$

30. Which line has an error in it?

$$\log_6 6 + \log_6 \sqrt{6} = x$$

1. $\log_6 6\sqrt{6} = x$

2. $6^x = 6\sqrt{6}$

3. $6^x = 6^1 \cdot 6^{\frac{1}{2}}$

4. $6^x = 6^{\frac{1}{2}}$

5. $x = \frac{1}{2}$

31. Which student solved for x correctly in the following problem? $2 \log x = 4$

Alice	Bao	Carlos	David
$2 \log x = 4$	$2 \log x = 4$	$2 \log x = 4$	$2 \log x = 4$
$\log x^2 = 4$	$\log x^2 = 4$	$\log x^2 = 4$	$\log x^2 = 4$
$x^2 = 4$	$x^2 = 4$	$x^2 = 10^4$	$x^2 = 10^4$
$x = 2$	$x = \pm 2$	$x^2 = 10000$	$x^2 = 10000$
		$x = 100$	$x = \pm 100$

32. Which student solved for x correctly in the following problem?

$$2 \log 3 + \log x = \log 36$$

Ariadna

$2 \log 3 + \log x = \log 36$
$\log 9 + \log x = \log 36$
$\log 9x = \log 36$
$9x = 36$
$x = 4$

Bella

$2 \log 3 + \log x = \log 36$
$\log 9 + \log x = \log 36$
$\log (9 + x) = \log 36$
$9 + x = 36$
$x = 27$

Choua

$2 \log 3 + \log x = \log 36$
$2(\log 3 + \log x) = \log 36$
$2 \log 3x = \log 36$
$\log 3x^2 = \log 36$
$3x^2 = 36$
$x^2 = 12$
$x = \sqrt{12}$

Domingo

$2 \log 3 + \log x = \log 36$
$2(\log 3 + \log x) = \log 36$
$2 \log 3x = \log 36$
$\log (3x)^2 = \log 36$
$9x^2 = 36$
$x^2 = 4$
$x = 2$

Math 3 Unit 9 Worksheet 7
Solving Exponential Equations without Common Bases

Name:_____

Date:_____Per:_____

Solve for x. Write your final answer in terms of logarithm(s) in two different ways. Then evaluate your answer using a scientific calculator and round your answer to the thousandth place.

1. $7^x = 12$

2. $5^x = 30$

3. $6^{x-1} = 92$

4. $2^{2x} = 74$

5. $4^{2x} = 22$

6. $4 \cdot 2^{3x} = 132$

7. $10^{x+1} = 180$

8. $5 \cdot 7^{x-3} = 200$

Solve for x. Write your final answer in terms of a logarithm. Then evaluate your answer using a scientific calculator and round your answer to the thousandth place.

9. $10^x = 12$

10. $e^x = 14$

11. $10^{x-1} = 129$

12. $e^{2x} = 71$

13. $2e^{x+5} = 22$

14. $10^{3x} - 7 = 4$

15. $3e^{5x} = 72$

16. $4 \cdot 10^{2x+7} = 800$

Solve each equation for *y* in terms of *x*.

17. $e^{y+4} = x^3 + 5$

18. $x^2 = \ln(4y - 1)$

19. $x^4 = 6 + 10^{2y-3}$

20. $5 + \log(3y + 2) = x - 7$

Choose the correct multiple choice response:

21. $\log_5 7 =$

   a. $\log 5 - \log 7$ b. $\log 7 - \log 5$ c. $7 \bullet \log 5$ d. $\frac{\log 7}{\log 5}$

22. $7^x = 14$

   a. $x = 2$ b. $x = \log 14$ c. $x = \frac{\log 14}{\log 7}$ d. $x = \log 2$

23. If $x = \log_4 15$ which is true about x?

   a. $x < 0$ b. $0 < x < 1$ c. $1 < x < 2$ d. $x > 2$

24. $\log_8 20 =$

   a. $\frac{\log 20}{\log 8}$ b. $\log\left(\frac{20}{8}\right)$ c. $\log 20 - \log 8$ d. $20\log 8$

25. $e^x = 4$

   a. $x = \log 4$ b. $x = \ln 4$ c. $x = \ln e^4$ d. $x = 4$

26. $2^x + 1 = 13$

   a. $x = \log_2 13 - 1$ b. $x = 6$ c. $x = \frac{\log 12}{\log 2}$ d. $x = \log 6$

1. A person invests $500 in an account that earns a yearly interest rate of 4%.
a) How much would this investment be worth in 10 years if the compounding frequency was once per year? Show the calculations you use.

b) How much would this investment be worth in 10 years if the compounding frequency was four times per year (quarterly)? Show the calculations you use.

2. An investment with a rate of 5% is compounded at different frequencies. Give the effective yearly rate, accurate to two decimal places, for each of the following compounding frequencies. Show your calculations.
a) Quarterly   b) Monthly   c) Daily

3. How much would $1000 invested at a 2% yearly rate, compounded monthly, be worth in 20 years?

4. A person invests $350 in a bank account that promises a rate of 2% compounded continuously.
a) Write an equation for the amount this investment would be worth after t-years.

b) How much would the investment be worth after 20 years?

c) Determine the time it will take for the investment to reach $400. Round to the nearest tenth of a year.

5. An investment of $500 is made at 2.8% interest compounded quarterly.
a) Write an equation that models the amount A the investment is worth t-years.

b) How much is the investment worth after 10 years?

c) Determine the time it will take for the investment to reach $800. Round to the nearest tenth of a year.

6. The value of an initial investment of $400 at 3% interest compounded quarterly can be modeled using which of the following equations, where t is the number of years since the investment was made?
(A) $A = 400(1.0075)^{4t}$ (B) $A = 400(1.03)^{4t}$ (C) $A = 400(1.0075)^{t}$ (D) $A = 400(1.0303)^{4t}$

7. Which of the following represents the value of an investment with a principal of $1500 with an interest rate of 2.5% compounded monthly after 5 years?
(A) $1,1697.11 (B) $4,178.22 (C) $1,699.50 (D) $5,168.71

[1-16] Evaluate the following logarithms. If needed, write an equivalent exponential equation. Do these without the use of your calculator.

1. $\log_6 36$

2. $\log_2 16$

3. $\log_2 0.25$

4. $\log_5 \left(\frac{1}{125}\right)$

5. $\log_3 \left(\frac{1}{27}\right)$

6. $\log_7(49\sqrt{7})$

7. $\log_3(-9)$

8. $\log_5 \sqrt{25}$

9. $\log_{27} 3$

10. $\log_9 1$

11. $\log_2 \frac{1}{8}$

12. $\log 10,000$

13. $\log \left(\frac{1}{100}\right)$

14. $\ln e^5$

15. $\ln \left(\frac{1}{e^4}\right)$

16. $\log_3 \sqrt[8]{27}$

[17 - 19] Find which two consecutive integers the logarithm lies between.

17. $\log_3 30$

18. $\log_4 21$

19. $\log_5 18$

[20-22] Find the *y*-intercept, if it exists, for the following functions.

20. $y = \log_3(x + 9) - 5$ 21. $y = \log_2(x + 1) + 3$ 22. $y = \log(x - 2) - 4$

23. Find the average rate of change for the function, $f(x) = \log_2(x + 1) - 5$, over the interval, $0 \le x \le 15$.

24. Find the average rate of change for the function, $f(x) = \log_3(x + 1) - 5$, over the interval, $0 \le x \le 8$.

Solve for *x*.

25. $9^x = \dfrac{1}{27}$ 26. $8^{2+x} = 2$ 27. $4^{1-x} = 8$

28. $27^{2x-1} = 3$ 29. $4^{3x+5} = 16^{x+1}$ 30. $3^{-(x+5)} = 9^{4x}$

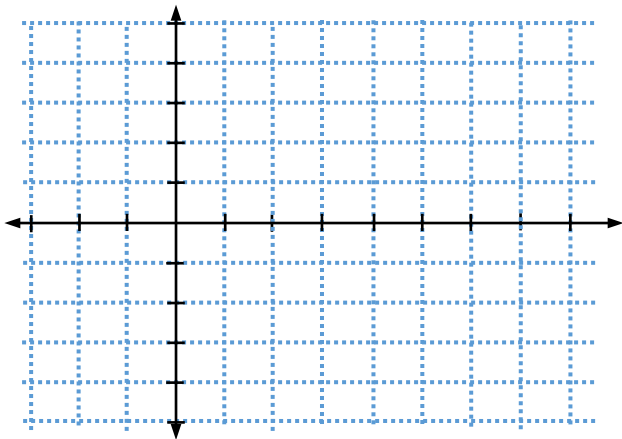31. $25^{2x} = 5^{x+6}$ 32. $6^{x+1} = 36^{x-1}$ 33. $\log_5(2x - 7) = 0$
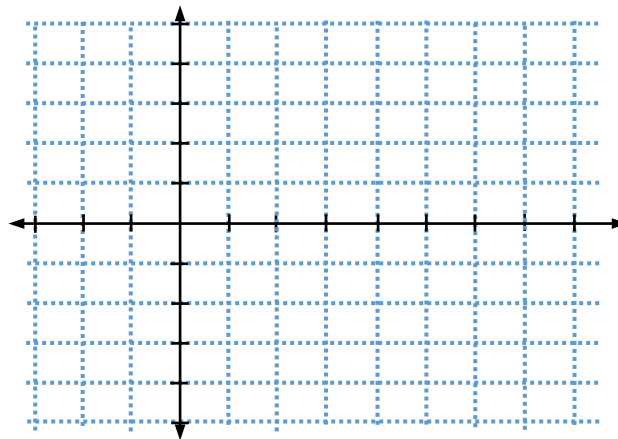
34. $\log_\pi x = 3$ 35. $\log_{64} 32 = x$ 36. $\log_8 x = \frac{2}{3}$

[37-38] For the following logarithmic functions: graph, find the x- and y-intercept(s), state the domain and range and find the equation of the vertical asymptote.

37. $y = \log_4 x$

x-intercept:_____

y-intercept:_____

Domain:_____

Range:_____

Vertical Asymptote:_____

38. $y = \log_2(x + 1) + 3$

x-intercept:_____

y-intercept:_____

Domain:_____

Range:_____

Vertical Asymptote:_____

39. Describe the translation from $f(x)$ to $g(x)$, given $f(x) = \log_3 x$ & $g(x) = \log_3(x - 2) + 5$

40. Describe the translation from $f(x)$ to $g(x)$, given $f(x) = \log_4 x$ & $g(x) = \log_4(x + 1) - 9$

Math 3 Unit 9   Review Worksheet

[41-44]: Completely expand and simplify each expression.

41. $\log 100xy^2$

42. $\ln \frac{e^5}{7}$

43. $\log_2 8x^2y\sqrt[3]{z}$

44. $\ln \frac{x\sqrt{y}}{z^2}$

[45-62]: Solve for x. Answer are to be integers or reduced fractions.

45. $\log_4 x = \frac{1}{2}\log_4 9 + \log_4 5$

46. $4\ln 2 = \ln 8x$

47. $\log x + \log(x-3) = 1$

48. $\log_2 x - \log_2(x+5) = \log_2 6$

49. $\log_3 x + \log_3(x+4) = \log_3 12$

50. $\log_7(7x+6) - \log_7 x = 2$

51. $\log_2 5 + \log_2(x+3) = 4$

52. $\log_3(x+8) + \log_3 x = 2$

[53-56]: Solve for x. Scientific calculators are needed. Round answers to nearest thousandth.

53. $9^x = 500$

54. $4^{x-1} + 10 = 52$

55. $5e^{3x-1} + 4 = 84$

56. $e^{3x} + 8 = 90$

57. A person invests \$3,000 in a bank account that pays at an annual interest rate of 2% compounded continuously. How much time will it take for the initial investment to double in value?
{Use the formula: $A = Pe^{rt}$}