

Math 3 Unit 4: Rational Functions

Unit	Title	Standards
4.1	Equivalent Rational Expressions	A.APR.6
4.2	Multiplying and Dividing Rational Expressions	A.APR.7
4.3	Adding and Subtracting Rational Expressions	A.APR.7
4.4	Solving Rational Equations	A.REI.2
4.5	Applied Problems with Rational Equations	A.REI.2
4.6	Graphing Rational Functions	F.IF.4, F.IF.5, F.IF.7d
Unit 4 Review		

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Math 3 Unit 4: Online Resources

4.1	Equivalent Rational Expressions	<ul style="list-style-type: none"> Patrick JMT: Rational Expressions - Writing in Lowest Terms – Ex 1 http://bit.ly/41erea Patrick JMT: Rational Expressions - Writing in Lowest Terms – Ex 2 http://bit.ly/41ereb Purple Math: Rational Expressions: More Simplifying http://bit.ly/41erec or http://bit.ly/41ered Khan Academy: Intro to Rational Expression Simplification http://bit.ly/41eree
4.2	Multiplying and Dividing Rational Expressions	<ul style="list-style-type: none"> Khan Academy: Multiplying & Dividing Rational Expressions: Monomials http://bit.ly/42mdrea Khan Academy: Multiplying Rational Expressions http://bit.ly/42mdreb Khan Academy: Dividing Rational Expressions http://bit.ly/42mdrec Patrick JMT: Rational Expressions - Multiplying and Dividing http://bit.ly/42mdred or http://bit.ly/42mdree or http://bit.ly/42mdref Purple Math: Dividing Rational Expressions http://bit.ly/42mdreg
4.3	Adding and Subtracting Rational Expressions	<ul style="list-style-type: none"> Khan Academy: Adding & Subtracting Rational Expressions – Like Denominators http://bit.ly/43asrea Khan Academy: Adding & Subtracting Rational Expressions – Unlike Denominators http://bit.ly/43asreb Patrick JMT: Rational Expressions - Adding and Subtracting. Ex 1 http://bit.ly/43asrec Patrick JMT: Rational Expressions - Adding and Subtracting. Ex 2 http://bit.ly/43asred Purple Math: Adding and Subtracting Rational Expressions http://bit.ly/43asree
4.4	Solving Rational Equations	<ul style="list-style-type: none"> Patrick JMT: Solving Rational Equations http://bit.ly/44srea Purple Math: Solving Rational Expressions http://bit.ly/44sreb Khan Academy: Solving Equation with Rational Expressions http://bit.ly/44srec Khan Academy: Solving Equation with Rational Expressions – Extraneous Solutions http://bit.ly/44sred Patrick JMT: Rational Expressions - Solving Rational Expressions – No Solution http://bit.ly/44sree
4.5	Applied Problems with Rational Equations	<ul style="list-style-type: none"> Khan Academy: Rational Equations Word Problem: Combined Rates http://bit.ly/45aprea or http://bit.ly/45apreb or http://bit.ly/45aprec Purple Math: "Work" Word Problems http://bit.ly/45apred
4.6	Graphing Rational Functions	<ul style="list-style-type: none"> Patrick JMT: Graphing Some Basic Rational Functions http://bit.ly/46grfa Algebra dotcom: Graphing Rational Functions http://bit.ly/46grfb

Unit 4 Worksheet 1
Equivalent Rational Expressions

Name: _____
Date: _____ **Per:** _____

[1-17] Find an equivalent rational expression in lowest terms, and identify the value(s) of the variable that must be excluded.

1. $\frac{16n}{20n}$

2. $\frac{x^3y}{y^4x}$

3. $\frac{30a^5b^3c^4}{24a^2b^3c^9}$

4. $\frac{db+dc}{db}$

5. $\frac{x^2-9}{x^2-2x-3}$

6. $\frac{3n^2-5n-2}{2n-4}$

7. $\frac{f(x)-2}{[f(x)]^2-4}$ if $f(x) = 3x$

8. $\left(\frac{g}{f}\right)(n)$ if $f(n) = 4n$ and $g(n) = 2n - 8n^2$

9. $\frac{4a^2-12a}{a^2-6a+9}$

10. $\frac{y-4}{4-y}$

11. $\frac{4x-2y}{3y-6x}$

12. $\left(\frac{f}{g}\right)(a)$ if $f(a) = a^2 - 49$ and $g(a) = 7 + a$

13. $\frac{f(x)}{[g(x)]^3}$ if $f(x) = 9 - x^2$ and $g(x) = x - 3$

14. $\frac{x^2-5x+6}{8-2x-x^2}$

15. $\frac{a-4}{2a^2-8a-3a+12}$

16. $\frac{y^2-3y+5y-15}{y^3-27}$

17. $\frac{8x^3-125}{4x^2-25}$

18. Write a rational expression with denominator $6b$ that is equivalent to

a) $\frac{a}{b}$

b) one-half of $\frac{a}{b}$

c) $\frac{1}{3}$

19. Remember that algebra is just another way to perform arithmetic, but with variables replacing numbers.

a) Simplify the following rational expression: $\frac{(x^2y)^2(xy)^3z^2}{(xy^2)^2yz}$.

b) Simplify the following rational expression without using a calculator: $\frac{12^2 \cdot 6^3 \cdot 5^2}{18^2 \cdot 15}$.

c) How are the calculations in parts (a) and (b) similar? How are they different? Which expression was easier to simplify?

Unit 4 Worksheet 2
Multiplying and Dividing Rational Expressions

Name: _____
Date: _____ Per: _____

[1-14] Multiply or divide the following and then simplify the result.

1. $\frac{15x}{14y} \cdot \frac{21y^2}{25x^3}$

2. $\frac{18x^2}{y} \div \frac{6x}{y}$

3. $\frac{7x}{2y^3} \cdot \frac{16y}{21x^4}$

4. $\frac{2a^2b}{3bc} \div \frac{9b^2c}{16ad^2}$

5. $\frac{x+2}{x^2-4} \div \frac{x+3}{x-2}$

6. $\frac{4x+8}{x^2-25} \cdot \frac{x-5}{5x+10}$

7. $\frac{x^2-6x-16}{x^2+4x-21} \div \frac{x^2+9x+14}{x^2-8x+15}$

8. $\frac{x^2-8x+12}{x^2-16} \cdot \frac{4x+16}{x^2-4x+4}$

$$9. \frac{\frac{x^2-9x-10}{x^2+x-6}}{\frac{x^2-1}{x^2-4}}$$

$$10. \frac{\frac{x^2+6x}{6x^2+15x}}{\frac{x^2-36}{2x^2+7x+5}}$$

$$11. \frac{2x^2+x-6}{x^2-2x-8} \cdot \frac{2x^2-x-3}{x^2-3x-4}$$

$$12. \frac{9-x^2}{x^2+6x+9} \div \frac{3x-9}{3x+9}$$

$$13. \frac{x^2-x}{2x^2+13x-7} \cdot \frac{2x^2+5x-3}{x^2+2x-3}$$

$$14. \frac{x^2+11x+18}{2x^2-50} \cdot \frac{x^2+6x+5}{2x^2+4x}$$

15. Suppose that $x = \frac{t^2+3t-4}{3t^2-3}$ and $y = \frac{t^2+2t-8}{2t^2-2t-4}$, for $t \neq 1$, $t \neq -1$, $t \neq 2$, and $t \neq -4$. Show that the value of x^2y^{-2} does not depend on the value of t .

16. Determine which of the following numbers is larger without using a calculator, $\frac{15^{16}}{16^{15}}$ or $\frac{20^{24}}{24^{20}}$.

(Hint: We can compare two positive quantities a and b by computing the quotient $\frac{a}{b}$. If $\frac{a}{b} > 1$, then $a > b$.

Likewise, if $0 < \frac{a}{b} < 1$, then $a < b$.)

17. One of two numbers can be represented by the rational expression $\frac{x-2}{x}$, where $x \neq 0$ and $x \neq 2$.

a. Find a representation of the second number if the product of the two numbers is 1.

b. Find a representation of the second number if the product of the two numbers is 0.

Unit 4 Worksheet 3
Adding and Subtracting Rational Expressions

Name: _____
Date: _____ **Per:** _____

[1-15] Perform the indicated operation. Write your final answer in simplest form.

1. $\frac{1}{3x} + \frac{5}{3x}$

2. $\frac{26}{7x^2y} - \frac{5}{7x^2y}$

3. $\frac{3x-4}{x+3} - \frac{6x+5}{x+3}$

4. $\frac{x^2+5x+16}{x+3} - \frac{1-3x}{x+3}$

5. $\frac{3x}{2x-6} + \frac{9}{6-2x}$

6. $\frac{x}{x-2} - \frac{x+1}{2-x}$

7. $\frac{2x^2}{x^2-9} - \frac{x+15}{9-x^2}$

8. $\frac{5}{3x^2} + \frac{x}{2}$

9. $\frac{1}{6xy} - \frac{2}{15x^2}$

10. $\frac{2}{x+3} - \frac{1-x}{x-4}$

11. $\frac{3}{x+2} - \frac{8}{x-2}$

12. $\frac{5}{x-2} + \frac{3}{x^2-4}$

EXTENSION:

30. Suppose that $x \neq 0$ and $y \neq 0$. We know from our work in this section that $\frac{1}{x} \cdot \frac{1}{y}$ is equivalent to $\frac{1}{xy}$. Is it also true that $\frac{1}{x} + \frac{1}{y}$ is equivalent to $\frac{1}{x+y}$? Provide evidence to support your answer.

31. Suppose that $x = \frac{2t}{1+t^2}$ and $y = \frac{1-t^2}{1+t^2}$. Show that the value of $x^2 + y^2$ does not depend on the value of t .

32. Show that for any real numbers a and b , and any integers x and y so that $x \neq 0$, $y \neq 0$, $x \neq y$, and $x \neq -y$,

$$\left(\frac{y}{x} - \frac{x}{y}\right) \left(\frac{ax + by}{x + y} - \frac{ax - by}{x - y}\right) = 2(a - b).$$

33. Suppose that n is a positive integer.

a. Simplify the expression $\left(1 + \frac{1}{n}\right) \left(1 + \frac{1}{n+1}\right)$.

b. Simplify the expression $\left(1 + \frac{1}{n}\right) \left(1 + \frac{1}{n+1}\right) \left(1 + \frac{1}{n+2}\right)$.

c. Simplify the expression $\left(1 + \frac{1}{n}\right) \left(1 + \frac{1}{n+1}\right) \left(1 + \frac{1}{n+2}\right) \left(1 + \frac{1}{n+3}\right)$.

d. If this pattern continues, what is the product of n of these factors?

Unit 4 Worksheet 4
Solving Rational Equations

Name: _____
Date: _____ **Per:** _____

[1-8] Solve the following equations and then state whether each equation has **no real solution**, **one real solution**, **two real solutions**, or **infinitely many solutions**.

1. $\frac{3}{x+2} = \frac{6}{x-1}$

2. $\frac{x}{2x+7} = \frac{x-5}{x-1}$

3. $\frac{f(x)}{[f(x)]^2-8} = \frac{2}{f(x)}$ if $f(x) = x$

4. $\frac{2}{3x+1} = \frac{6}{9x+3}$

5. $\frac{x^2-3}{x+2} = \frac{x-3}{2}$

6. $\frac{x-4}{x+3} = \frac{x}{3}$

7. $\frac{-3}{x+1} = \frac{4}{x-1}$

8. $\frac{6}{x-3} - \frac{6}{x} = \frac{18}{x^2-3x}$

[9-17] Solve the following equations and check for extraneous solutions.

9. $\frac{2}{x+1} + \frac{1}{x+1} = \frac{4}{x}$

10. $\frac{x}{x+2} + \frac{2}{x^2+5x+6} = \frac{5}{x+3}$

$$11. \frac{3}{f(x)} = \frac{f(x)+3}{f(x)} - 2 \quad \text{if } f(x) = x - 3$$

$$12. \frac{4x+1}{x+1} = \frac{12}{x^2-1} + 3$$

$$13. \frac{f(x)+1}{f(x)} - \frac{2}{f(x)+1} = \frac{x-f(x)}{f(x)} \quad \text{if } f(x) = x - 1$$

$$14. \frac{5x}{x-2} = \frac{3x+4}{x-2}$$

$$15. \frac{x}{x^2-1} + \frac{2}{x+1} = \frac{1}{2x-2}$$

$$16. \frac{2}{x^2-x} = \frac{1}{x-1}$$

$$17. \frac{x}{x-2} - \frac{2}{x-4} = \frac{4x-12}{x^2-6x+8}$$

18. Given $f(x) = \frac{5}{x+2}$ and $g(x) = \frac{3}{x}$. Solve the equation $f(x) = g(x)$ algebraically.

19. Given $f(x) = \frac{7}{x-5}$ and $g(x) = \frac{4}{x-2}$. Solve the equation $f(x) - g(x) = 0$ algebraically.

20. Given $f(x) = \frac{x}{x+3}$ and $g(x) = \frac{8}{x+6}$. Solve the equation $(f - g)(x) = 0$ algebraically.

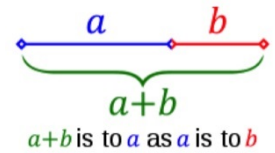
21. Given $f(x) = \frac{2}{x+1}$ and $g(x) = \frac{x-6}{x^2-1}$. Solve the equation $f(x) = g(x)$ algebraically.

22. Create and solve a rational equation that has 0 as an extraneous solution.

23. Create and solve a rational equation that has 2 as an extraneous solution.

EXTENSION:

1. Two lengths a and b , where $a > b$, are in *golden ratio* if the ratio of $a + b$ is to a is the same as a is to b . Symbolically, this is expressed as $\frac{a}{b} = \frac{a+b}{a}$. We denote this common ratio by the Greek letter *phi* (pronounced "fee") with symbol φ , so that if a and b are in common ratio, then $\varphi = \frac{a}{b} = \frac{a+b}{a}$. By setting $b = 1$, we find that $\varphi = a$ and φ is the positive number that satisfies the equation $\varphi = \frac{\varphi+1}{\varphi}$. Solve this equation to find the numerical value for φ .



2. Remember that if we use x to represent an integer, then the next integer can be represented by $x + 1$.
- a. Does there exist a pair of consecutive integers whose reciprocals sum to $\frac{5}{6}$? Explain how you know.
- b. Does there exist a pair of consecutive integers whose reciprocals sum to $\frac{3}{4}$? Explain how you know.
- c. Does there exist a pair of consecutive even integers whose reciprocals sum to $\frac{3}{4}$? Explain how you know.
- d. Does there exist a pair of consecutive even integers whose reciprocals sum to $\frac{5}{6}$? Explain how you know.

Unit 4 Worksheet 5
Applied Problems with Rational Equations

Name: _____
Date: _____ **Per:** _____

1. Aubrey can wash all the windows of a retail store in 6 hours. Maxwell can wash all the windows of the same retail store in 9 hours.
 - a) Write an equation that can be used to find the time t , in hours, it would take Aubrey and Maxwell to wash all the windows of the retail store together.

 - b) Solve the equation for t that you wrote in part a)

2. Der can wash all the dishes in the house in 20 minutes. Her brother Yashua can wash all the dishes in the house in 30 minutes.
 - a) Write an equation that can be used to find the time t , in minutes, it would take Der and Yashua to wash all dishes in the house together.

 - b) Solve the equation for t that you wrote in part a)

3. A group of friends decide to evenly divide the \$72 cost of watching a premiere boxing match on pay-per-view TV. Initially, there are x friends, but then 3 friends decide not to watch the boxing match and spend their money to see, Star Wars: Episode 23: The Neverending Force, at the movie theatre instead. This causes each remaining friend to have to each pay \$2 more.
 - a) Create an equation that represents the situation and can be used to solve for x .

 - b) Use the equation you created in part a) to solve for x , the initial number of friends.

4. A group of college students rent a large house to live in and agree to evenly divide the \$1200 monthly rent. Initially, there are n college students, but then 5 additional college students decide to join the group, causing each college student to pay \$40 less per month.
 - a) Create an equation that represents the situation and can be used to solve for n .

 - b) Use the equation you created in part a) to solve for n the initial number of college students.

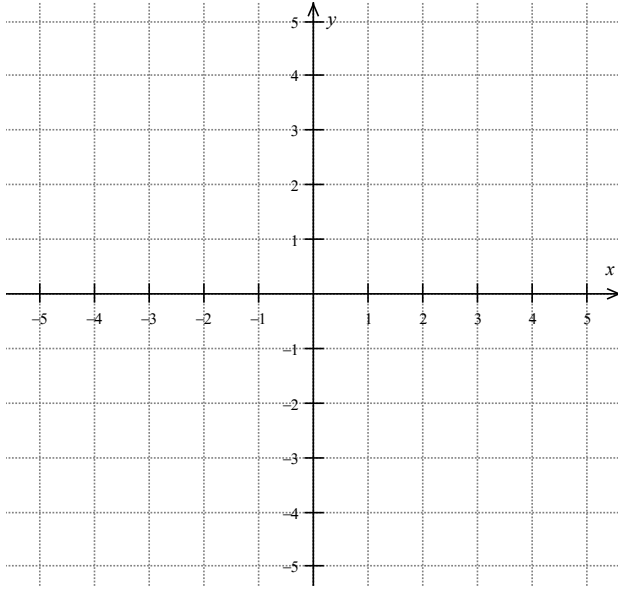
5. The density, D , of an object is defined as $D = \frac{M}{V}$ where M represents the mass of the object, and V represents the volume of the object. Solve this equation for V .
6. Suppose a basketball player has a shooting percentage, P , that is inversely proportional to the square root of her distance from the basket, d according to the equation $P = \frac{k}{\sqrt{d}}$ where k is the proportionality constant. Solve this equation for d .
7. Joule's Law states that $P = \frac{E^2}{R}$ where P is the power in watts, E is the voltage in volts, and R is the resistance in ohms. Solve this equation for E .
8. Newton's law of universal gravitation, $F = \frac{Gm_1m_2}{r^2}$, measures the force of gravity between two masses m_1 and m_2 , where r is the distance between the centers of the masses, and G is universal gravitational constant. Solve this equation for G .
9. Consider the rational equation $\frac{1}{R} = \frac{1}{x} + \frac{1}{y}$. Solve this equation for R and simplify.
10. Consider an ecosystem of rabbits in a park that starts with 10 rabbits and can sustain up to 60 rabbits. An equation that roughly models this scenario is $P(t) = \frac{60}{1 + \frac{5}{t+1}}$, where $P(t)$ represents the rabbit population in year t of the study.
- What is the rabbit population in year 10? Round your answer to the nearest whole rabbit.
 - Solve this equation for t . Describe what this equation represents in the context of this problem.
 - At what time does the population reach 50 rabbits?

Unit 4 Worksheet 6
Graphing Rational Functions

Name: _____
 Date: _____ Per: _____

1. $f(x) = \frac{1}{x+2}$

a) Sketch the graph of f on the xy -plane given.



b) Find the y -intercept of f .

c) Find the x -intercept of f .

d) State the domain of f .

e) State the range of f .

f) State the equation(s) of any vertical asymptotes of f .

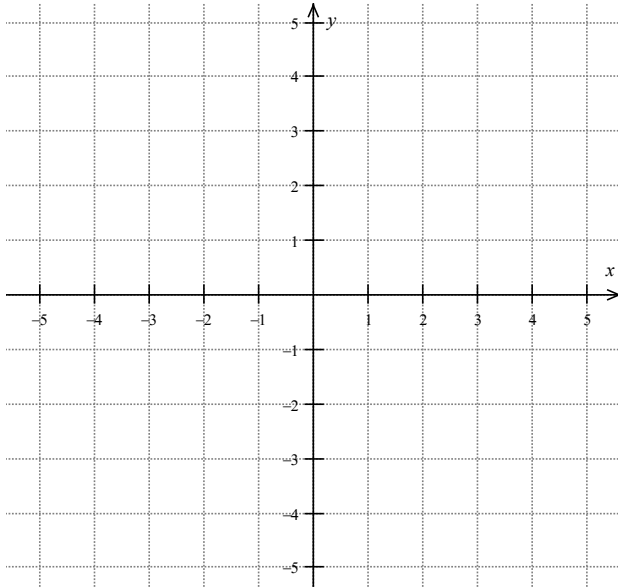
g) State the equation(s) of any horizontal asymptotes of f .

h) State the interval(s) on which the graph of f is increasing?

i) State the interval(s) on which the graph of f is greater than zero?

2. $g(x) = \frac{1}{x} - 2$

a) Sketch the graph of g on the xy -plane given.



b) Find the y -intercept of g .

c) Find the x -intercept of g .

d) State the domain of g .

e) State the range of g .

f) State the equation(s) of any vertical asymptotes of g .

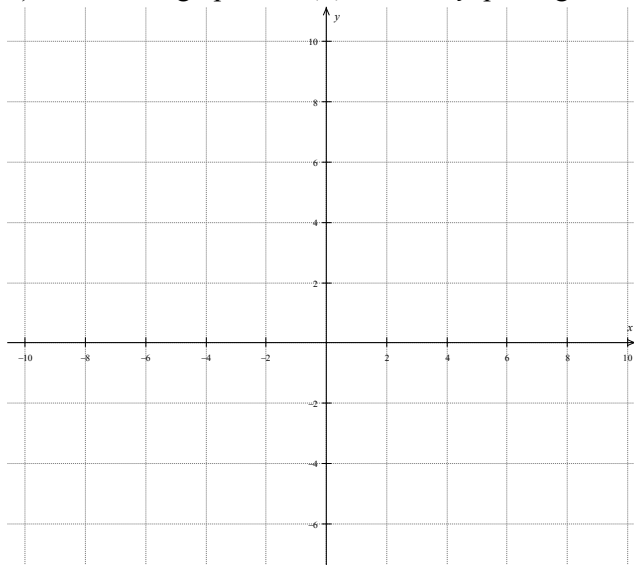
g) State the equation(s) of any horizontal asymptotes of g .

h) State the interval(s) on which the graph of g is decreasing?

i) State the interval(s) on which the graph of g is greater than zero?

3. Let $h(x) = g(f(x))$ with $g(x) = \frac{1}{x} + 1$ and $f(x) = x - 3$ a) Find $h(x)$

b) Sketch the graph of $h(x)$ on the xy -plane given. c) Find the y -intercept of h .



d) Find the x -intercept of h .

e) State the domain of h .

f) State the range of h .

g) State the equation(s) of any vertical asymptotes of h .

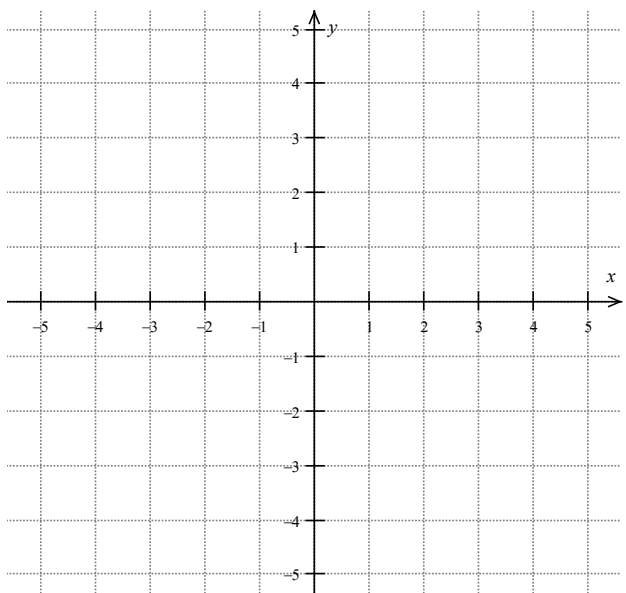
h) State the equation(s) of any horizontal asymptotes of h .

i) State the interval(s) on which the graph of h is increasing?

j) State the interval(s) on which the graph of h is less than zero?

4. Let $j(x) = f(g(x))$ with $f(x) = \frac{-1}{x} - 3$ and $g(x) = x + 1$ a) Find $j(x)$

b) Sketch the graph of j on the xy -plane given. c) Find the y -intercept of j .



d) Find the x -intercept of j .

e) State the domain of j .

f) State the range of j .

g) State the equation(s) of any vertical asymptotes of j .

h) State the equation(s) of any horizontal asymptotes of j .

i) State the interval(s) on which the graph of j is decreasing?

j) State the interval(s) on which the graph of j is less than zero?

For problems 5-8, state the equations of any horizontal and vertical asymptotes of each function.

5. $f(x) = \frac{-2}{x-1}$

6. $g(x) = \frac{3}{x} + 5$

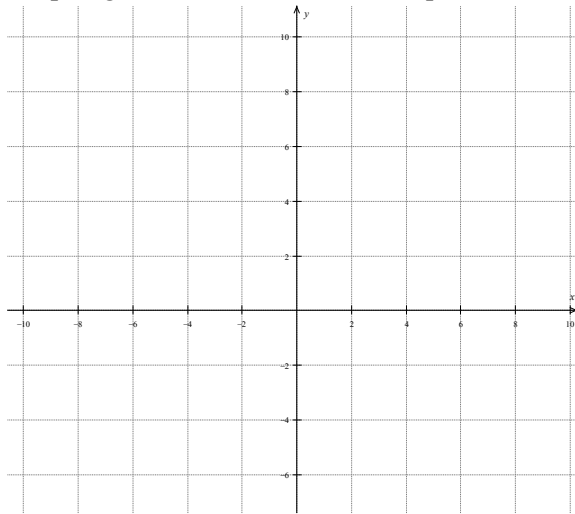
7. $h(x) = \frac{-5}{x+4} + 2$

8. $j(x) = \frac{2}{x-7} - 4$

9. Given $f(x) = \frac{12}{x}$ and $g(x) = 6$

a) Solve the equation $f(x) = g(x)$ by

Graphing $f(x)$ and $g(x)$ on the xy -plane below.

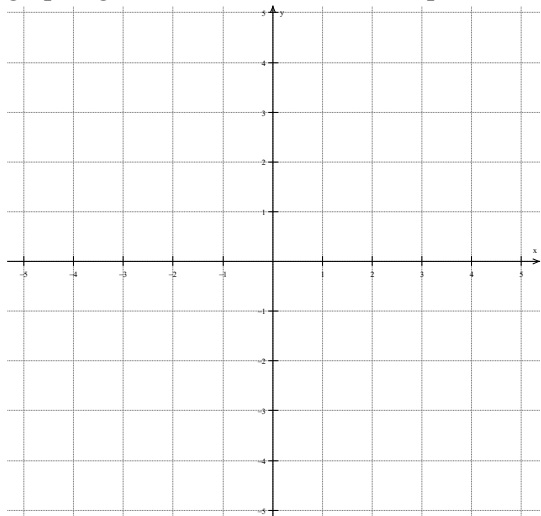


b) Solve the equation $f(x) = g(x)$ algebraically.

c) How are the intersection points on the graph related to the algebraic solutions?

10. Given $f(x) = \frac{3}{x}$ and $g(x) = 3$

a) Solve the equation $(f - g)(x) = 0$ by graphing $f(x)$ and $g(x)$ on the xy -plane below.



b) Solve the equation $(f - g)(x) = 0$ algebraically.

c) Explain the relationship between solving an equation graphically and solving algebraically.

Math 3 Unit 4 Review Worksheet
Rational Expressions and Functions

Name: _____
Date: _____ Per: _____

Simplify and state any excluded values.

1. $\frac{2x-8}{4-x}$

2. $\frac{x^2-5x}{x^2-7x+10}$

3. $\frac{2x^2-32}{x^3+6x^2+8x}$

4. $\frac{x^3+3x^2-x-3}{2x^3+5x^2-3x}$

Multiply or divide and simplify completely.

5. $\frac{3x}{4y} \cdot \frac{9y^2}{12x^3}$

6. $\frac{7x}{12yz^2} \div \frac{49x^3}{18y^2z^3}$

7. $\frac{x^2-3x-10}{x^2-4x+4} \cdot \frac{4x-8}{2x-10}$

8. $\frac{2x^2-18}{x^2-x-2} \div \frac{10x-30}{x^2+x-6}$

9. $\frac{4-x^2}{x+2} \cdot \frac{12x}{3x-6}$

10. $\frac{\frac{3x^2+30x+75}{x-5}}{\frac{x}{x^2-25}}$

11. $\frac{3x+2}{5x^2-x} \cdot \frac{10x^2+3x-1}{6x^2+x-2}$

12. $\frac{\frac{x^3-6x^2+8x}{x^2-8x+16}}{\frac{2x-4}{10x^2-40x}}$

Simplify each complex fraction.

$$25. \frac{x - \frac{1}{x}}{1 + \frac{1}{x}}$$

$$26. \frac{\frac{4}{x} + \frac{2}{3}}{\frac{x+3}{5} + \frac{1}{x}}$$

$$27. \frac{1 + \frac{3}{x}}{\frac{1}{2} - \frac{x}{2x^2}}$$

Solve for x . Then state whether the equation has no real solutions, one real solution, two real solutions, or infinitely many real solutions.

$$28. \frac{6}{x-2} = \frac{5}{x-3}$$

$$29. \frac{x}{x-4} = \frac{4}{x+4}$$

$$30. \frac{2}{x-2} = \frac{x}{4}$$

$$31. \frac{2x+2}{6} = \frac{x+1}{3}$$

Solve for x . Check for extraneous solutions.

$$32. \frac{9}{f(x)-2} - \frac{12}{f(x)} = \frac{3f(x)-6}{f(x)} \text{ if } f(x) = x + 2$$

$$33. \frac{4}{x-3} + 6 = \frac{10}{x-3}$$

$$34. \frac{1}{x-1} + \frac{1}{x+2} = \frac{3}{x^2+x-2}$$

$$35. x + \frac{x-1}{x-3} = \frac{2}{x-3}$$

$$36. \frac{2x}{x-1} + \frac{x-5}{x^2-1} = 1$$

$$37. \frac{4}{x^2-8x+12} = \frac{x}{x-2} + \frac{1}{x-6}$$

38. Given $f(x) = \frac{6}{x-2}$ and $g(x) = \frac{5}{x-3}$. Solve the equation $(f - g)(x) = 0$ algebraically.

39. A group of students decides to purchase a billboard for their school that costs \$120 and to divide the cost evenly among the students. Initially there are x students, but then nine more students decide to join the group, causing each student to pay \$3 less.

a) Create an equation that represents the situation and can be used to solve for x , the initial number of students.

b) Use the equation you created in part a) to solve for x , the initial number of students.

40. Chet can change all the tires on a truck in 10 minutes. Destiny can change all the tires on the same truck in 15 minutes.

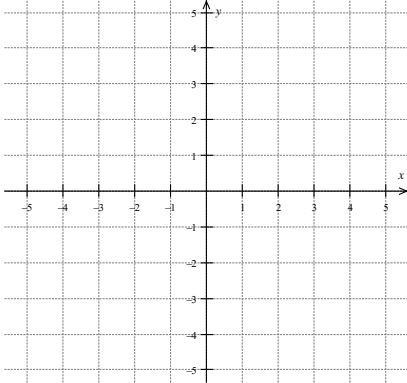
a) Write an equation that can be used to find the time t , in minutes, it would take Chet and Destiny to change all the tire on the truck if they work together.

b) Solve the equation for t that you wrote in part a)

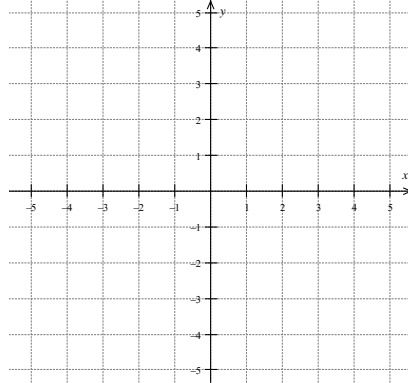
For problems 41 – 43, do all of the following:

- a) Sketch the graph of f .
- b) Find the y - intercept of f .
- c) Find the x - intercept of f .
- d) State the domain of f .
- e) State the range of f .
- f) State the equation(s) of any vertical asymptotes of f .
- g) State the equation(s) of any horizontal asymptotes of f .
- h) State the interval(s) on which the graph of f is increasing?
- i) State the interval(s) on which the graph of f is decreasing?
- j) State the interval(s) on which the graph of f is greater than zero?
- k) State the interval(s) on which the graph of f is less than zero

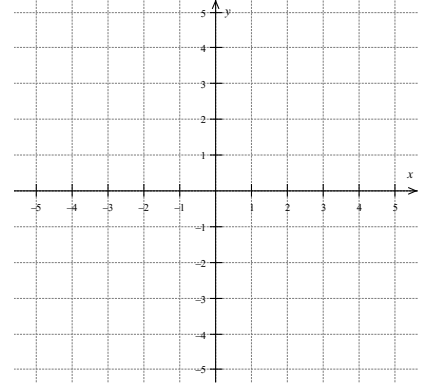
41. $f(x) = \frac{1}{x+3} - 2$



42. $f(x) = \frac{-1}{x-2} + 1$



43. $f(x) = \frac{-2}{x+1}$



For problems 44 and 45: If $f(x) = (g \circ h)(x)$, then

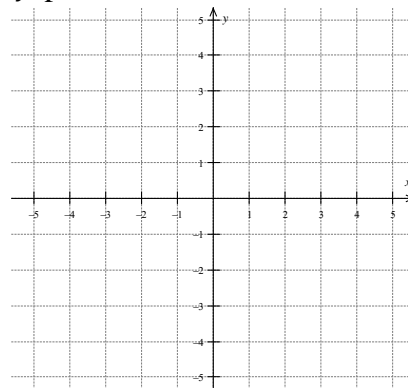
a) find $f(x)$ **and** **b)** state the equations for any horizontal and vertical asymptotes of $f(x)$.

44. $g(x) = \frac{3}{x} - 5$ & $h(x) = x - 2$

45. $g(x) = \frac{-2}{x} + 4$ & $h(x) = x + 1$

46. Given $f(x) = \frac{8}{x}$ and $g(x) = 4$,

a) Solve the equation $f(x) = g(x)$ by graphing $f(x)$ and $g(x)$ on the same xy -plane.



b) Solve the equation $f(x) = g(x)$ algebraically.

c) How are the intersection point(s) on the graph related to the algebraic solution(s)?